Homework 1

PHYS798S Spring 2016

Due Thursday, February 4, 2016

Homework Policy:

Your grade will be based on homework and a paper. In exchange for not giving exams, I ask that you do the homework. You may work on homework together, but not doing the homework will imperil your grade–I am willing to give bad grades if homework is not done. Please hand in your homework on time. I will not accept late homework, unless a valid excuse (such as illness) is given, preferably before the homework is due.

1. (a) Use the London equation to show that
$\overrightarrow{\nabla} \times \left(\overrightarrow{\nabla} \times \overrightarrow{B}\right) = -\frac{1}{\lambda^2}\overrightarrow{B}$ in a superconductor.

(b) Suppose a superconducting surface lies in the $y - z$ plane. A magnetic field is applied in the z direction parallel to the surface, $\overrightarrow{B} = (0, 0, B_0)$. Given that inside the superconductor the magnetic field is a function of x only, $\overrightarrow{B} = (0, 0, B_z(x))$ show that
$\frac{d^2 B_z(x)}{dx^2} = \frac{1}{\lambda^2} B_z(x)$.

(c) Solving the ordinary differential equation in (b) show that the magnetic field near a surface of a superconductor has the form $B = B_0 e^{-x/\lambda}$.

2. Screening in a superconducting slab. Solve the London equations for an infinite superconducting plate of finite thickness $2t$, assuming the magnetic field B_0 is applied parallel to both surfaces. Find both the magnetic field and the supercurrent inside the slab. As examples, plot the current and magnetic field for a thickness $2t = \lambda$, and $2t = 10\lambda$.

3. Two-fluid model. A more realistic model for a superconductor assumes that there is a density n_n of normal electrons which obey a Drude-like equation,
$\frac{dJ_n}{dt} = \frac{n_n q^2}{m} E - \frac{J_n}{\tau}$
as well as a density n_s of superelectrons which obey a London equation,
$\frac{dJ_s}{dt} = \frac{n_s q^2}{m} E$.

(a) Using the $e^{+i\omega t}$ time convention, find the frequency-dependent complex conductivity $\sigma(\omega)$. Assume that each 'fluid' responds independently to the electric field.

(b) What simple lumped-element circuit has an admittance $Y = 1/Z$ (where Z is the complex impedance of the circuit) with the same frequency dependence?

(c) Show that, in the low-frequency limit, the normal-fluid response is purely ohmic, while the superfluid response is purely inductive. In this limit, plot $\sigma_1(T)$ and $\sigma_2(T)$ vs T using the empirical relationships

$n_s(T) = n_0 \left[1 - (T/T_c)^4\right]; n_n(T) = n_0 - n_s(T),$

where n_o is the density of electrons in the material. The expression for $n_s(T)$ is a fairly good approximation for the superfluid density in a clean metal, but the second expression is seriously flawed: $n_s(T)+n_n(T)$ is not equal to the total electron density.