

**Physics 798S Spring16 Superconductivity**  
**Homework 6**  
**Due April 8, 2016**

1. The Ginzburg-Landau differential equations can be used *above*  $T_c$  by having  $\alpha(T)$  positive (it is negative below  $T_c$ ) and, since  $\Psi$  will be small above  $T_c$ , dropping the cubic term. Suppose that  $\Psi = \Psi_0$  at  $x = 0$ , and the material fills the region  $x > 0$ . Let  $\mathbf{A} = 0$ . Show that  $\Psi$  decays away from the boundary, with a characteristic length

$$\xi = [\hbar^2 / 2m^* \alpha(T)]^{1/2}$$

This is in contrast to the behavior below  $T_c$ , where spatially-extended order parameters are possible.

This situation occurs even when the “superconductor” has  $T_c = 0$ , in which case the coherence length is called  $\xi_N$ , the normal-metal coherence length. The boundary condition  $\Psi = \Psi_0$  at  $x = 0$  can be created by having the normal metal be in electrical contact with a superconductor. This is the origin of the *proximity effect*, where superconductivity is induced in a normal metal by proximity to a superconductor. The superconducting electrons have “leaked” from the superconductor into the normal metal.

2. A superconducting film is infinite in the  $y$  and  $z$  directions, and has thickness  $d$  in the  $x$  direction. It obeys the linear GL equation,

$$\frac{\partial^2 \psi}{\partial x^2} = - \frac{\psi}{\xi_{GL}^2(T)}$$

The left side of the film faces vacuum, and the right side is in contact with an infinite normal metal, so the boundary conditions are:

$$\left. \frac{\partial \psi}{\partial x} \right|_{x=0} = 0 \qquad \left. \frac{\partial \psi}{\partial x} \right|_{x=d} = - \frac{\psi}{b}$$

Find  $T_c^{\text{bilayer}}$  of the bilayer as a function of  $d$ . (You will have to solve a transcendental equation by graphical or other means. Choose the solution for  $\xi = \xi_0/[1 - T_c^{\text{bilayer}}/T_c^{\text{Bulk}}]^{1/2}$  that gives the highest  $T_c^{\text{bilayer}}$ .) Give explicit equations for  $d \gg b$  and  $d \ll b$ . In the latter case,  $T_c^{\text{bilayer}} = 0$  is not acceptable, the answer must include the next order term.

3. Consider a normal metal extending from  $-d/2$  to  $+d/2$  which is described by the linearized GL equation with  $\mathbf{A} = 0$ ,

$$\frac{\partial^2 \psi}{\partial x^2} = + \frac{\psi}{\xi_{GL}^2(T)}$$

Why is there a “+” sign in this equation compared to that in problem 2, above? The normal metal is sandwiched between two superconductors, which impose the boundary conditions:

$$\psi = \psi_L \quad \text{at } x = -d/2$$

$$\psi = \psi_R e^{i\gamma} \quad \text{at } x = +d/2.$$

Show that this leads to a solution in the normal metal which carries a supercurrent  $J_s$ , with  $J_s = J_c \sin \gamma$ , and find an explicit expression for  $J_c$ . How does  $J_c$  depend on  $d/\xi_N$ ? The equation for  $J_s$  was first derived by Josephson for tunnel junctions using microscopic theory, but applies to superconducting – normal – superconducting (SNS) junctions as well.