

Physics 798S
Superconductivity
Spring 2016
Homework 4
Due Thursday 3 March, 2016

1. Collapsing products of non-commuting operators

When checking the normalization of the BCS ground state wavefunction:

$$|\Psi_{G,BCS}\rangle = \prod_{k=k_1}^{k=k_M} (u_k + v_k c_{\vec{k}\uparrow}^+ c_{-\vec{k}\downarrow}^+) |0\rangle$$

, where the product is over all momentum states from k_I to k_M , one has to consider a double product over all momentum states when

$$\langle \Psi_{G,BCS} | \Psi_{G,BCS} \rangle$$

constructing . Show that this double product can be safely collapsed into a single product by carefully considering the anti-commuting properties of the Fermionic operators. We are interested in demonstrating this collapse in general, and use the normalization calculation simply to illustrate the point.

2. BCS Variational Calculation

Starting from the BCS pairing Hamiltonian in terms of the $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$, work through the variational calculation and derive the final results for $u_{\mathbf{k}}$, $v_{\mathbf{k}}$, and the zero temperature gap Δ .