Physics 798S
Superconductivity
Spring 2016
Homework 2
Due Tuesday, 16 February, 2016

1. The Schrödinger equation for the Macroscopic Quantum Wavefunction $\Psi(\mathbf{r},t)$ for a superconductor. $i\hbar\frac{\partial\Psi}{\partial t} = \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} - q^*A\right)^2\Psi + q^*\phi\Psi$

a) Under the assumption that the number density $n^*(\mathbf{r},t)$ is constant in space and time, derive the energy-phase relationship:

$$-\hbar\,\partial\theta/\partial t = (1/2n^*)\,\Lambda\,J_s^2 + q^*\,\phi$$

from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically.

b) Now assume that $n^*(r,t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:

$\partial n^*/\partial t = -\,\nabla\bullet(n^*\,\mathbf{v_s})$

Interpret this result physically (it may help to multiply both sides by q*).

2. Fluxoid quantization can be written as $\oint_C\left(\Lambda\vec{J}_s + \vec{A}\right)\cdot d\vec{l} = n\Phi_0$, where n is any positive or negative integer or zero. Using the expression for the supercurrent density in terms of the superfluid velocity, $\vec{J}_s = n^*q^*\vec{v}_s$, and the definition of the canonical momentum, $\vec{p}_{can} = m^*\vec{v}_s + q^*\vec{A}$, show that fluxoid quantization is an expression of the Bohr-Sommerfeld quantization condition: $\oint\vec{p}\cdot d\vec{q} = nh$, where (q, p) are conjugate coordinate and momentum, and h is Planck's constant.

3. A ring having 10 μm inner diameter and a film thickness of 1 μm is formed of a material having penetration depth $\lambda = 50$ nm at the temperature under consideration. Find the current density (in A/cm^2) at the inner surface of the ring when the ring contains one flux quantum.