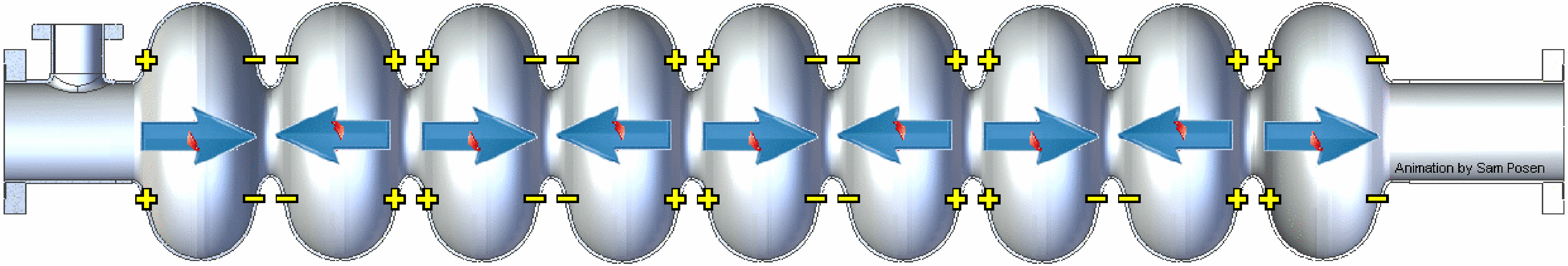
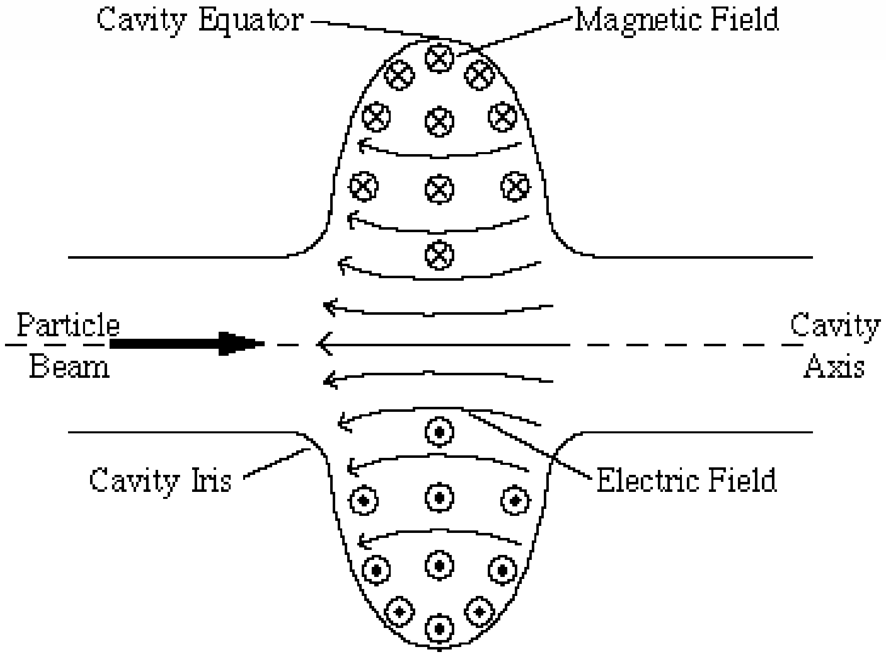
Superconducting Microwave Cavities and Resonators

Steven M. Anlage

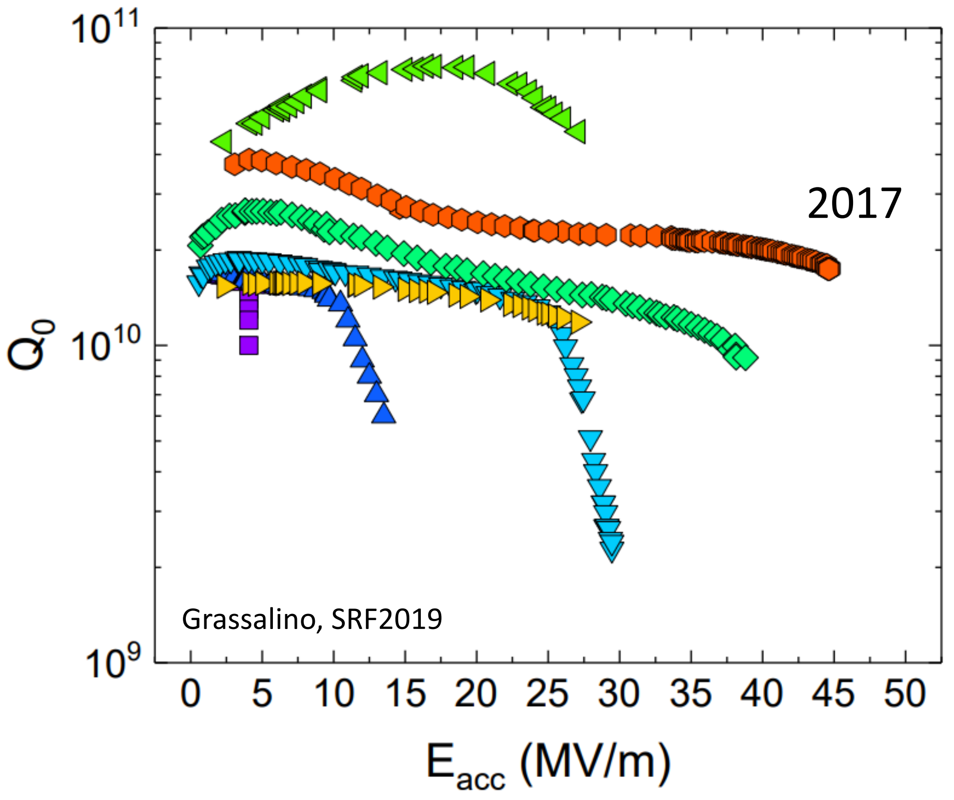
Quantum
Materials
Center

Superconducting Radio Frequency (SRF) Cavity

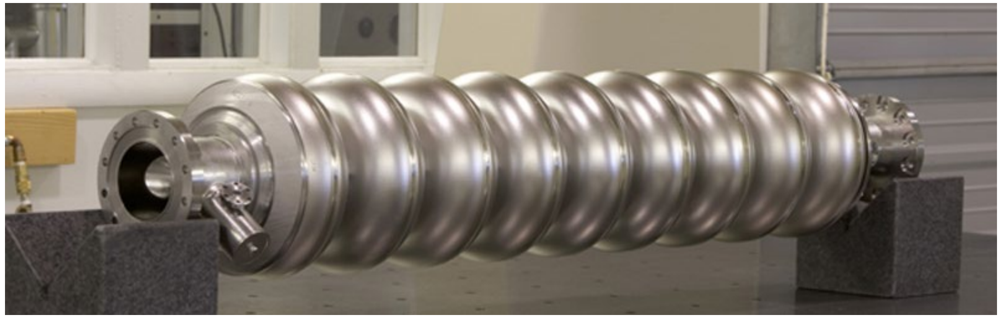




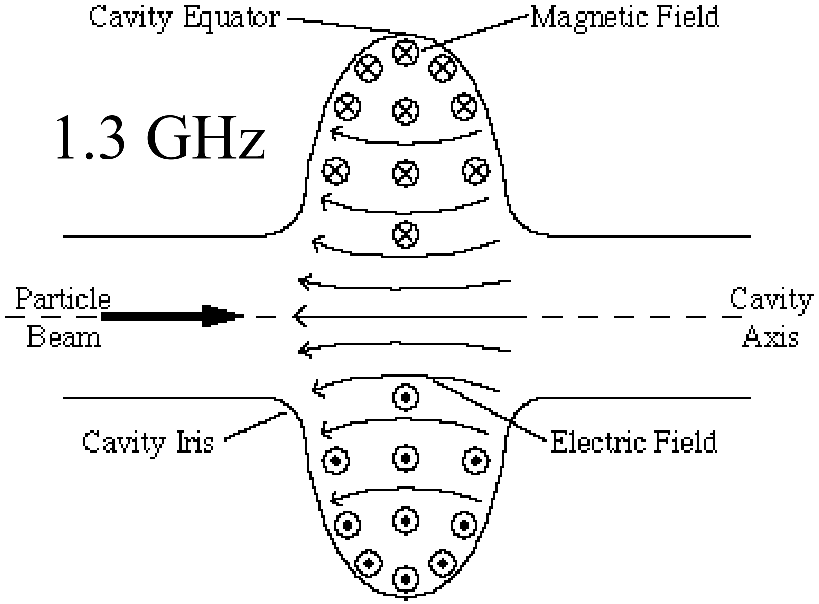
https://www.lhc-closer.es



Superconducting Radio Frequency Cavity

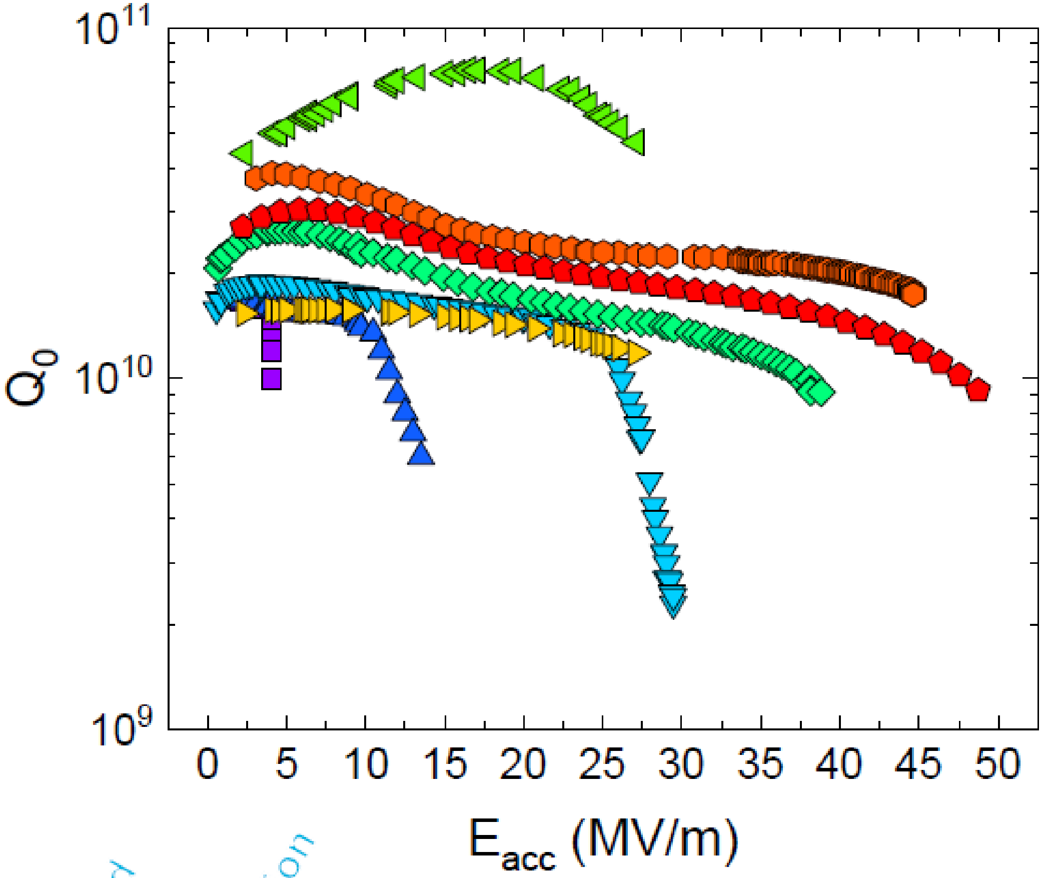


https://www-bd.fnal.gov/srf/about_NML.html



1.3 GHz

https://www.classe.cornell.edu/rsrc/Home/Research/SRF/SrfCavitiesAPrimerOne/SRFBas1.GIF



SRF2019 (Anna Grassellino)

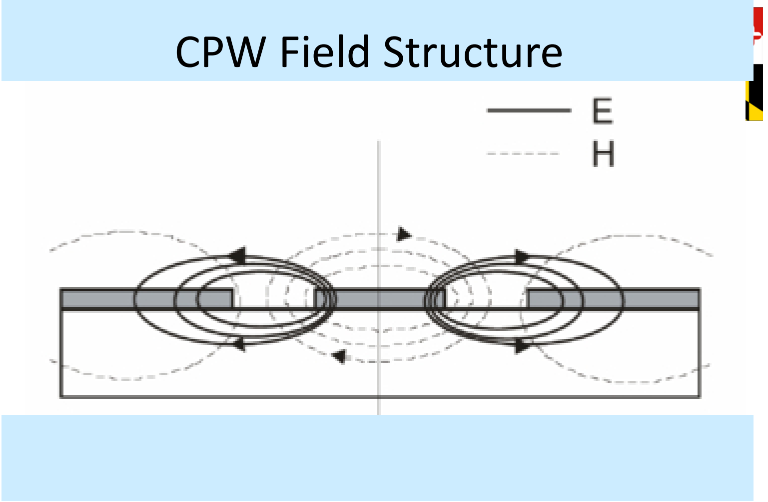
Superconducting Microwave Resonators for QC

- **Thin Film Resonators**
  - **Co-planar Waveguide**
  - **Lumped-Element**
  - **SQUID-based**

- **Bulk Resonators**

- **Coupling to Resonators**

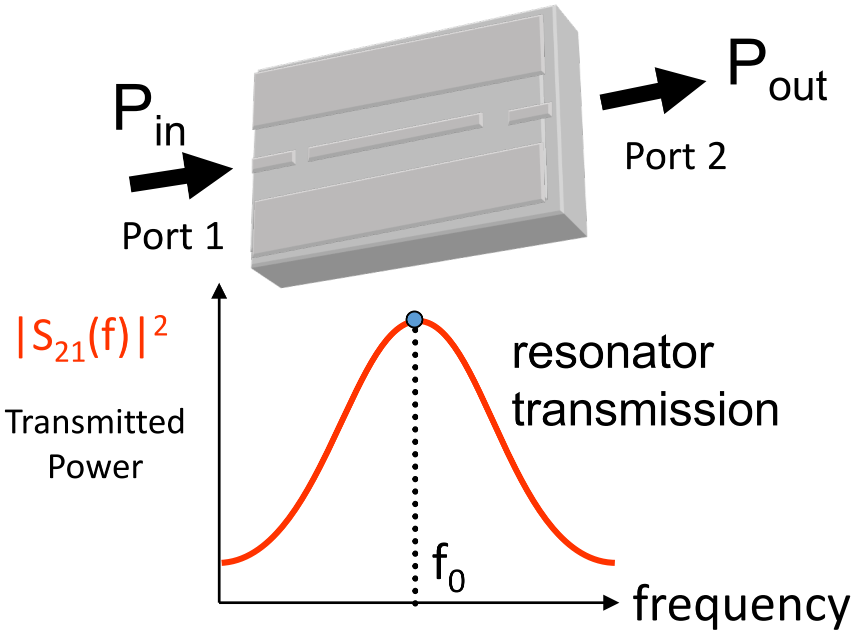
Resonators

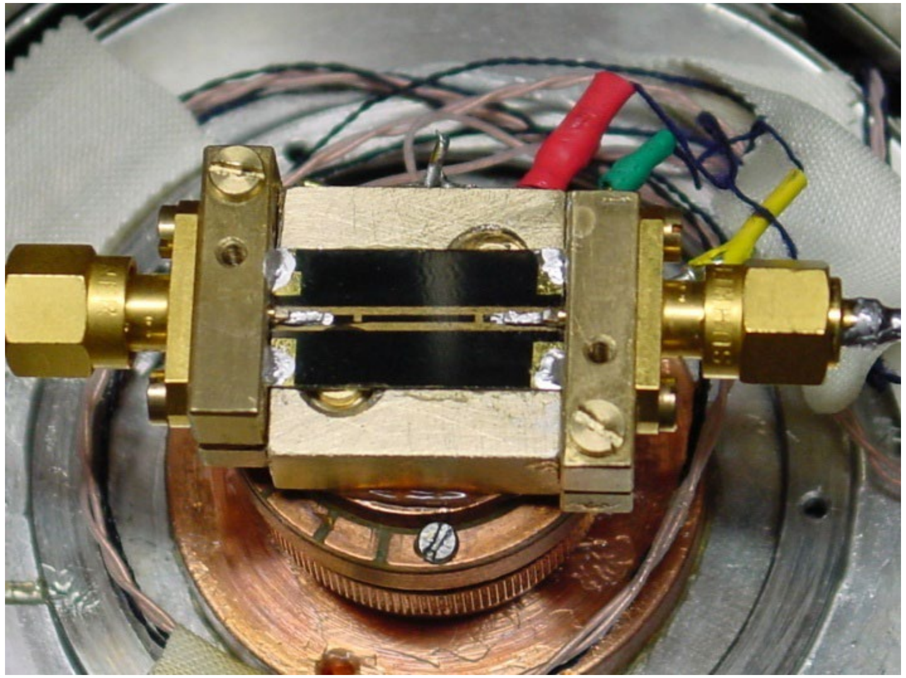


… the building block of superconducting applications …

Microwave surface impedance measurements
Cavity Quantum Electrodynamics of Qubits
Superconducting RF Accelerators
Metamaterials ($\mu_{eff} < 0$ ‘atoms’)
etc.

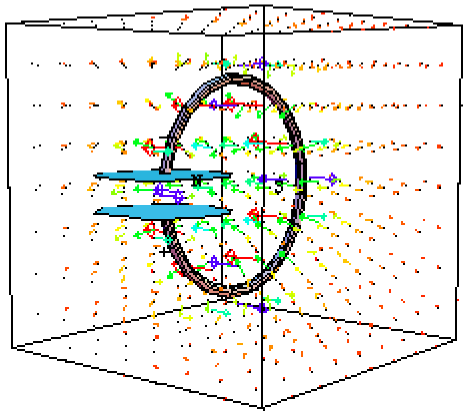
co-planar waveguide (CPW) resonator

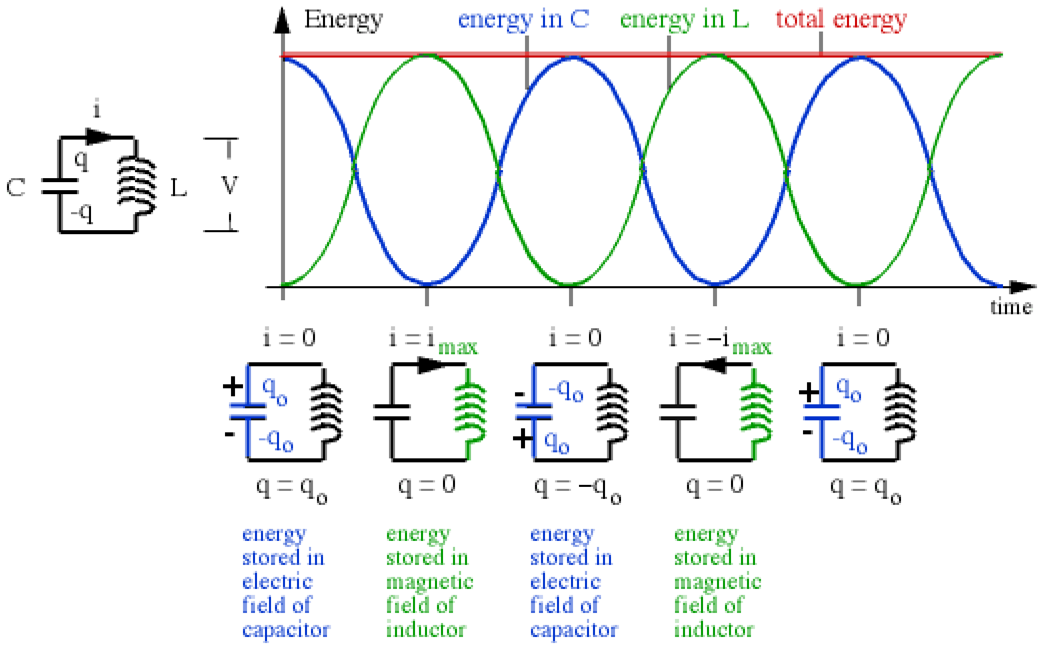




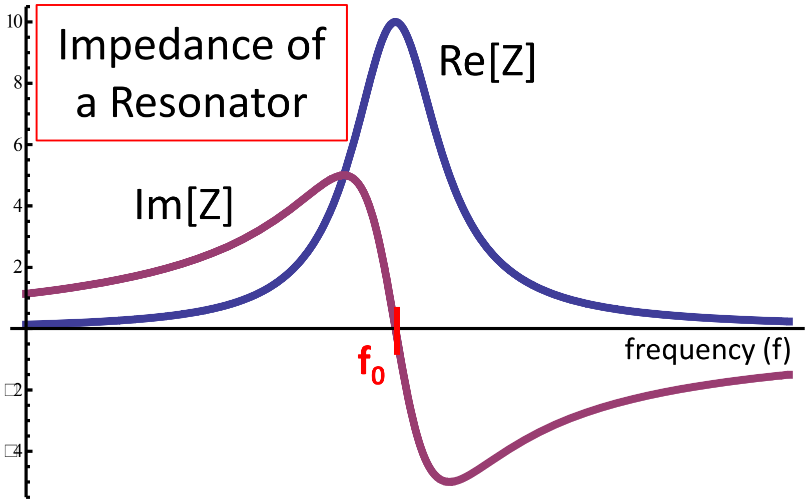
The Inductor-Capacitor Circuit Resonator

Animation link



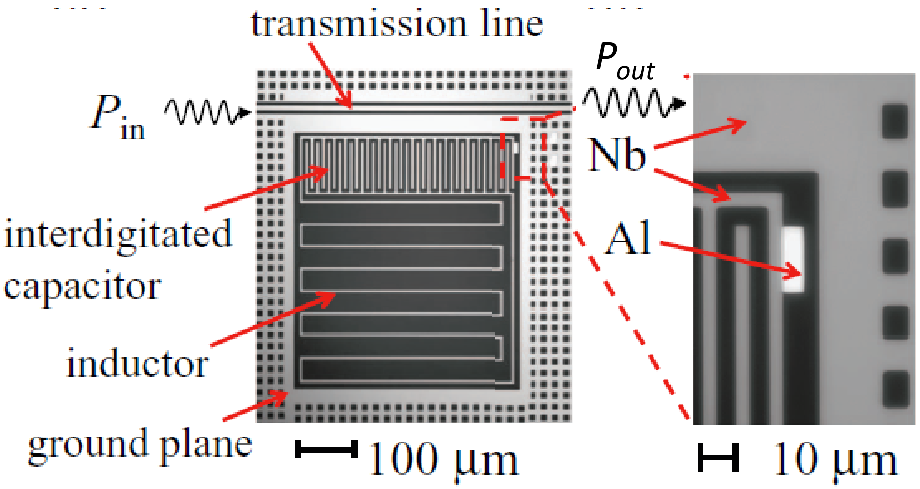


http://www.phys.unsw.edu.au

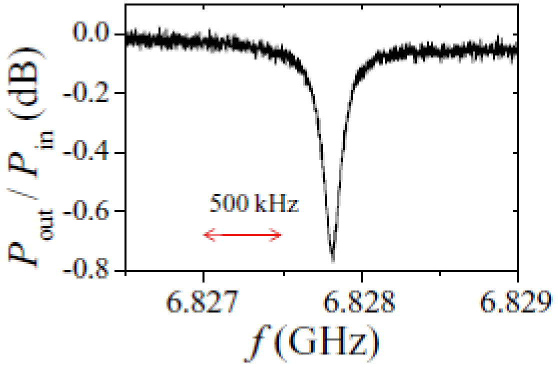


Im[Z] = 0 on resonance

Lumped-Element LC-Resonator

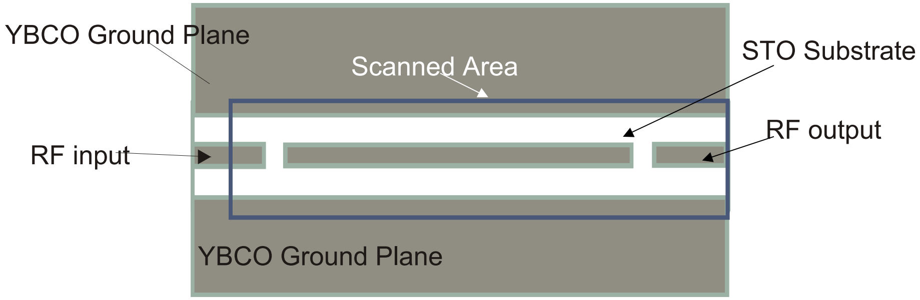


Z. Kim AIP ADVANCES 1, 042107 (2011)



Resonators (continued)

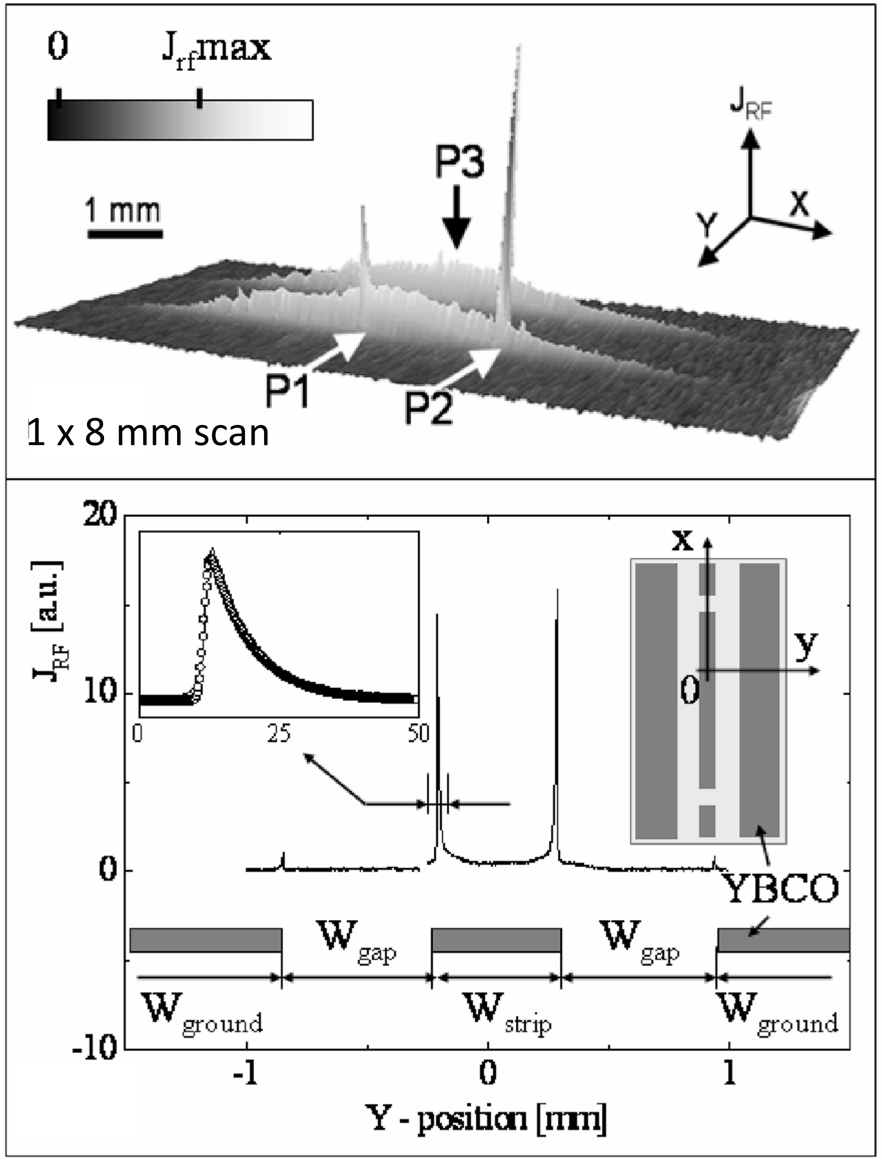
YBCO/LaAlO$_3$
CPW Resonator
Excited in Fundamental Mode
Imaged by Laser Scanning Microscopy*



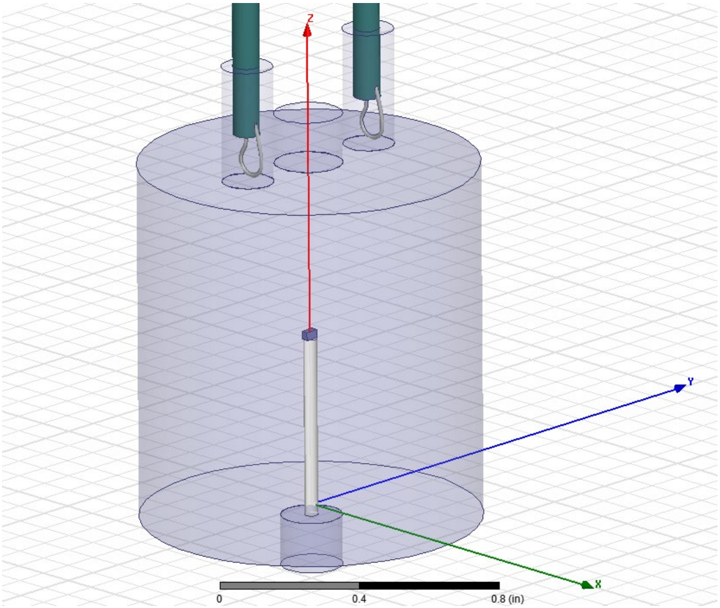
T = 79 K
P = - 10 dBm
f = 5.285 GHz
W_{strip} = 500 μm

[Trans. Line Resonator Detail]

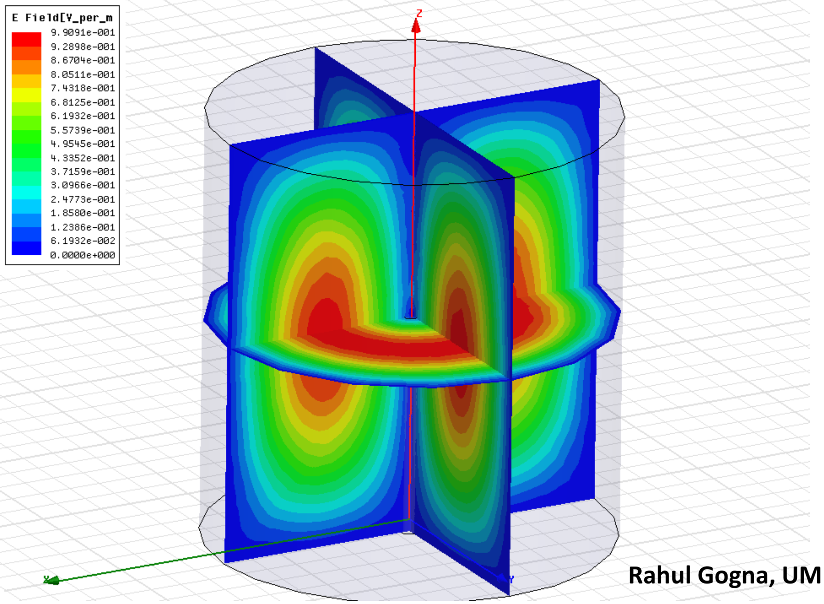
*A. P. Zhuravel, *et al.*, J. Appl. Phys. 108, 033920 (2010)
G. Ciovati, *et al.*, Rev. Sci. Instrum. 83, 034704 (2012)



Three-Dimensional Resonator

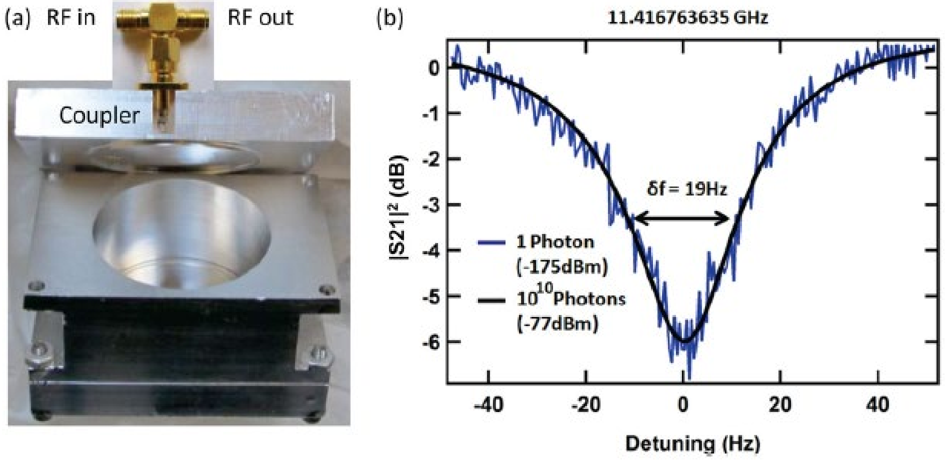


Inductively-coupled cylindrical cavity with sapphire “hot-finger”



Rahul Gogna, UMD

TE$_{011}$ mode Electric fields



Al cylindrical cavity

TE$_{011}$ mode

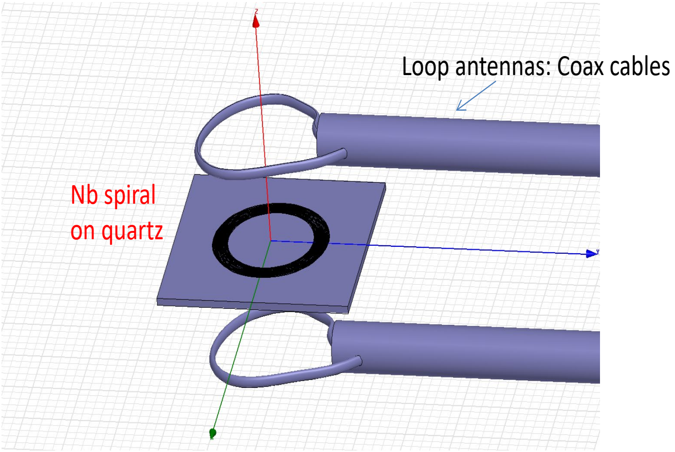
Q = 6 x 10^8

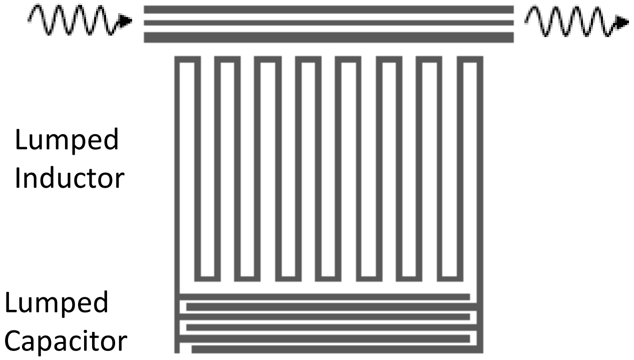
T = 20 mK

M. Reagor, APL 102, 192604 (2013)

Coupling to Resonators

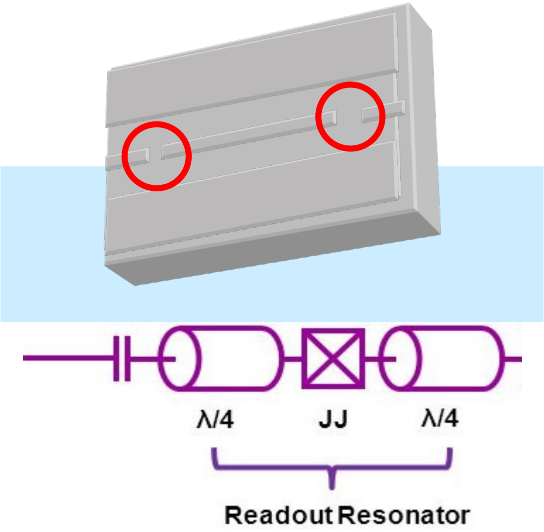
Inductive

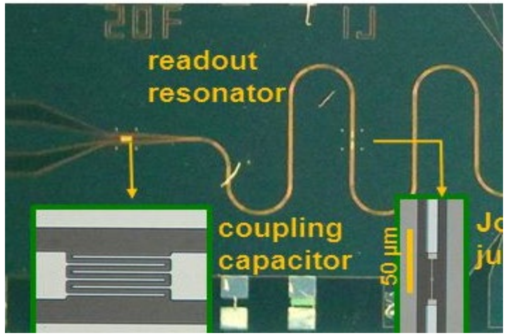




Lumped Element Kinetic Inductance Detector

Capacitive



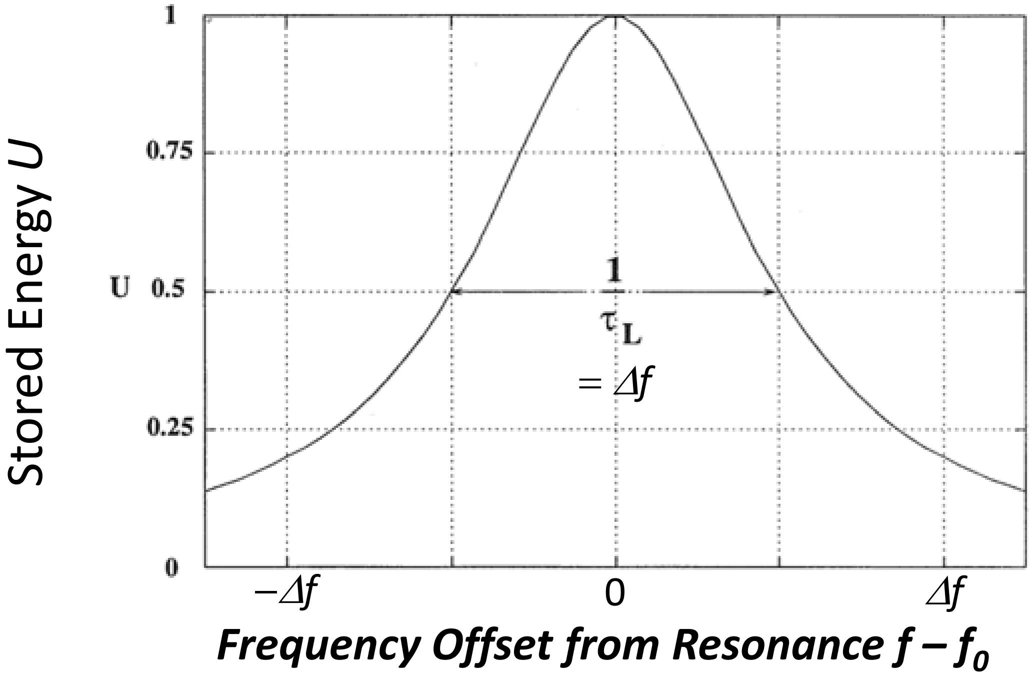


P. Bertet SPEC, CEA Saclay

Quality Factor

- Two important quantities characterise a resonator:

  The resonance frequency **f_0** and the quality factor **Q**



$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0 U}{P_c}$$

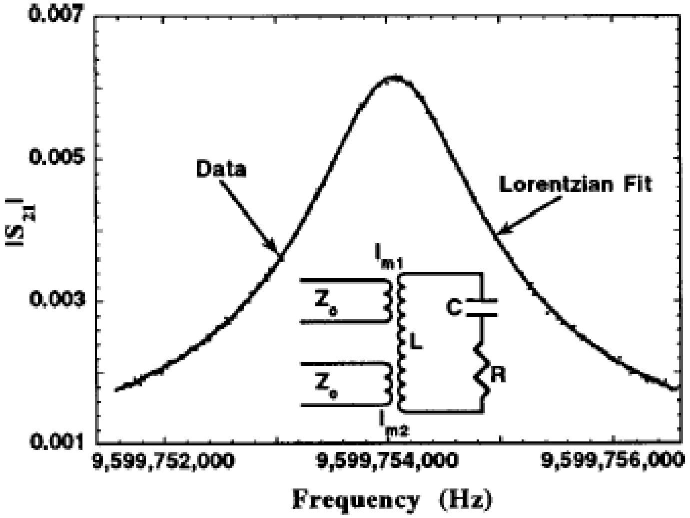
$$U = U_0 \exp(-t/\tau_L)$$

[Q of a shunt-coupled resonator Detail]

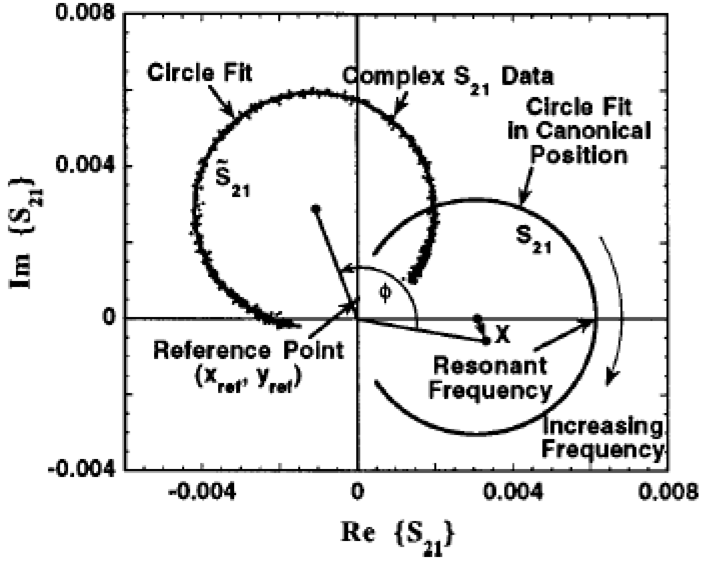
- Where U is the energy stored in the cavity volume and P_c/ω_0 is the energy lost per RF period by the induced surface currents

Some typical Q-values: SRF accelerator cavity Q ~ 10^{11} 3D qubit cavity Q ~ 10^{8}

Scattering Parameter of Resonators



$$|S_{21}(f)| = \frac{|\overline{S_{21}}|}{\sqrt{1+4Q^2\left(\frac{f}{f_0}-1\right)^2}}$$



$$S_{21}(f) = \frac{\overline{S_{21}}}{1+i2Q\left(\frac{f}{f_0}-1\right)}$$

$$\tilde{S}_{21} = (S_{21}+X)e^{i\phi}$$

JOURNAL OF APPLIED PHYSICS VOLUME 84, NUMBER 6 3392 15 SEPTEMBER 1998

Measurement of resonant frequency and quality factor of microwave resonators: Comparison of methods

Paul J. Petersan and Steven M. Anlage[a)]

Microwave Losses

- **Microscopic Sources of Loss**
  - **2-Level Systems (TLS) in Dielectrics**
  - **Flux Motion**

- **What Limits the Q of Resonators?**

Microwave Losses / 2-Level Systems (TLS) in Dielectrics

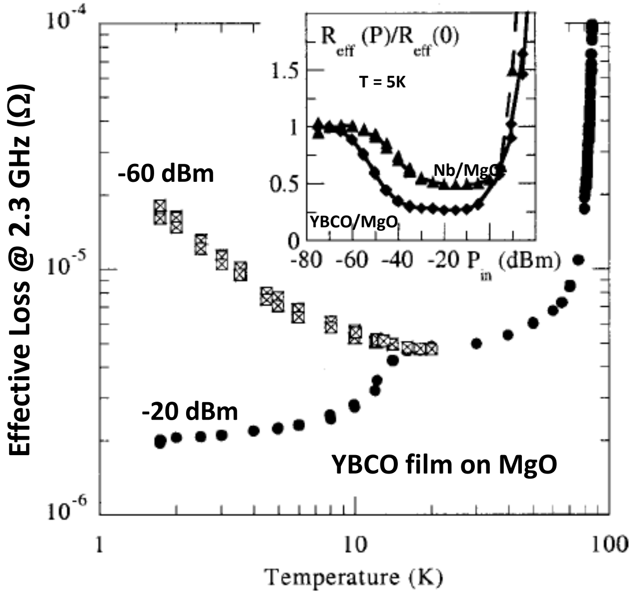
Classic reference:

Two-level states in glasses

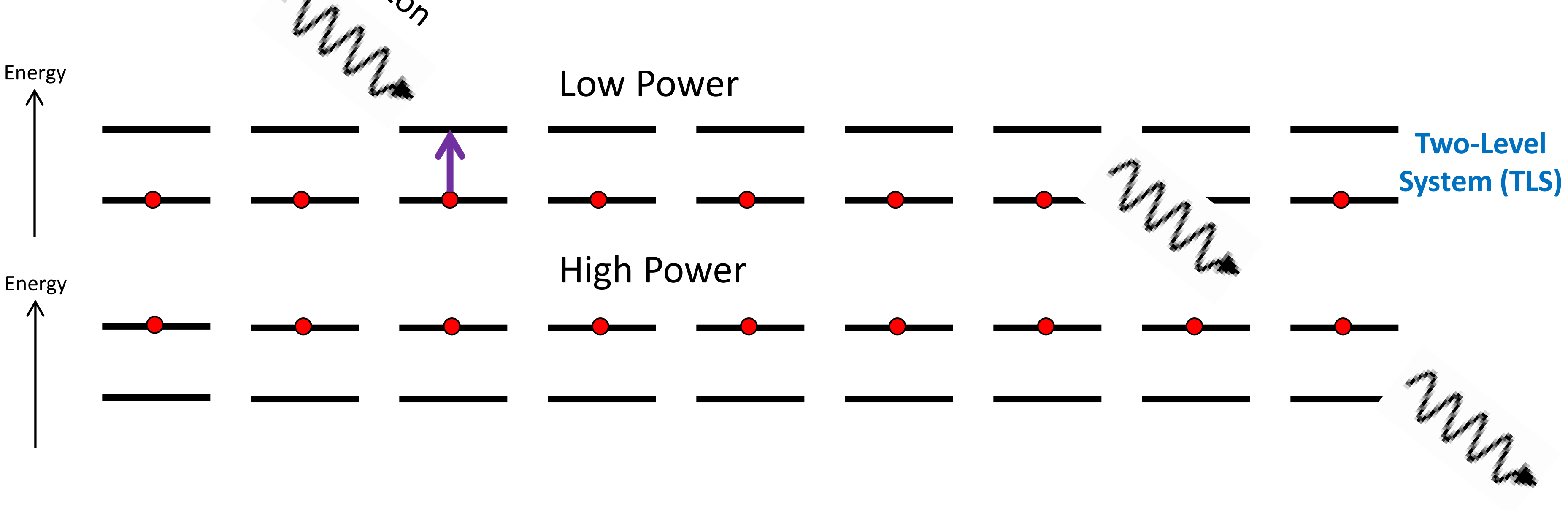
W A Phillips

Rep. Prog. Phys. **50** (1987) 1657-1708.

TLS in MgO substrates



M. Hein, APL 80, 1007 (2002)



Microwave Losses / Flux Motion

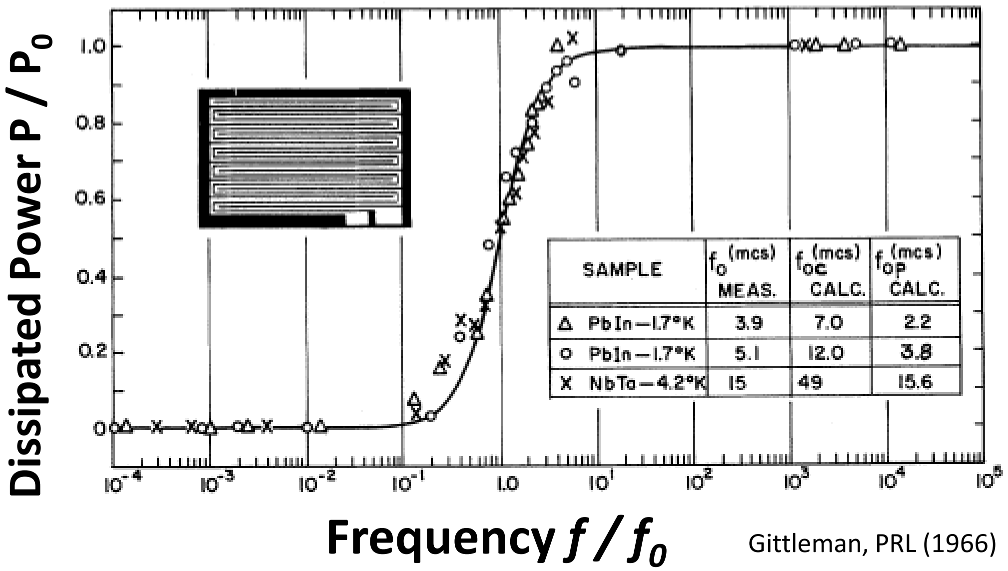
Single-vortex response to AC current (Gittleman-Rosenblum model)

$$m\ddot{\vec{x}} + \eta\dot{\vec{x}} + \vec{F}_{pin} = \vec{J} \times \vec{\Phi}$$

Equation of motion for vortex
in a rigid lattice (vortex-vortex force is constant)

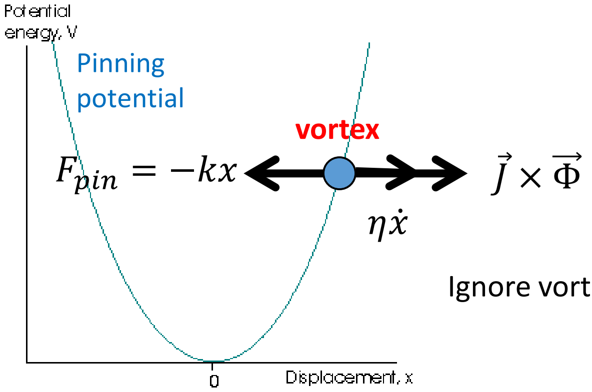
m Effective mass of vortex

η Vortex viscosity ~ σ_n



SAMPLE	f_0 (mcs) MEAS.	f_{oc} (mcs) CALC.	f_{op} (mcs) CALC.
△ PbIn—1.7°K	3.9	7.0	2.2
○ PbIn—1.7°K	5.1	12.0	3.8
X NbTa—4.2°K	15	49	15.6

Gittleman, PRL (1966)



Ignore vortex inertia

Pinning frequency f_0

$$2\pi\, f_0 \equiv k/\eta$$

What Limits the Q of Resonators?

$$Q \sim \frac{\text{Stored Energy}}{\text{Energy Dissipated per Cycle}}$$

<u>Assumption</u>: Loss mechanisms add linearly

$$\frac{1}{Q_{Total}} = \frac{1}{Q_{Ohmic}} + \frac{1}{Q_{Dielectric}} + \frac{1}{Q_{Coupling}} + \frac{1}{Q_{Radiation}} + \cdots$$

$$\frac{1}{Q_{Ohmic}} \sim \frac{R_s}{2} \oiint \left|H_{tang}\right|^2 dS$$

$$\frac{1}{Q_{Dielectric}} \sim \frac{\omega \epsilon''}{2} \oiiint \left|\vec{E}\right|^2 dV$$

$$\frac{1}{Q_{Coupling}} \sim$$ Power dissipated in load impedance

$$\frac{1}{Q_{Radiation}} \sim P_{Radiated}$$

Microwave Modeling and Simulation

- **Computational Electromagnetics**
  - **Finite Element Approach (FEM)**
  - **Finite Difference Time Domain (FDTD)**

- **Solvers**
  - **Eigenmode**
  - **Driven**
  - **Transient Time-Domain**

- **Uses**

Computational EM: FEM and FDTD

The Maxwell curl equations

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

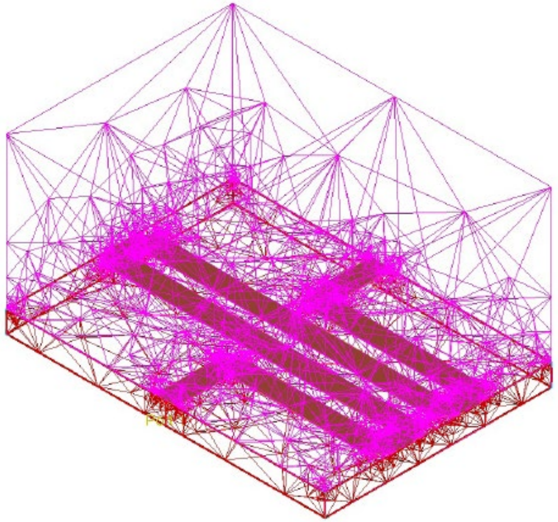
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D}$$

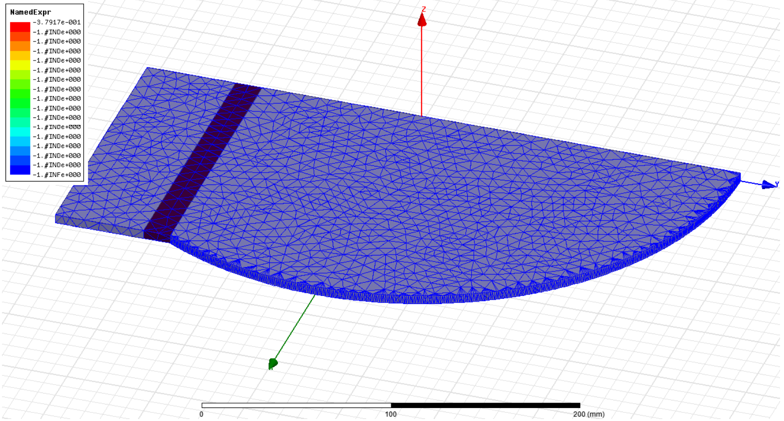
Finite Difference Time Domain (FDTD): Directly approximate the differential operators on a grid Staggered in time and space. E and H computed on a regular grid and advanced in time.

Finite-Element Method (FEM): Create a finite-element mesh (triangles and tetrahedra), expand the fields in a series of basis functions on the mesh, then solve a matrix equation that minimizes a variational functional corresponds to the solutions of Maxwell's equations subject to the boundary conditions.

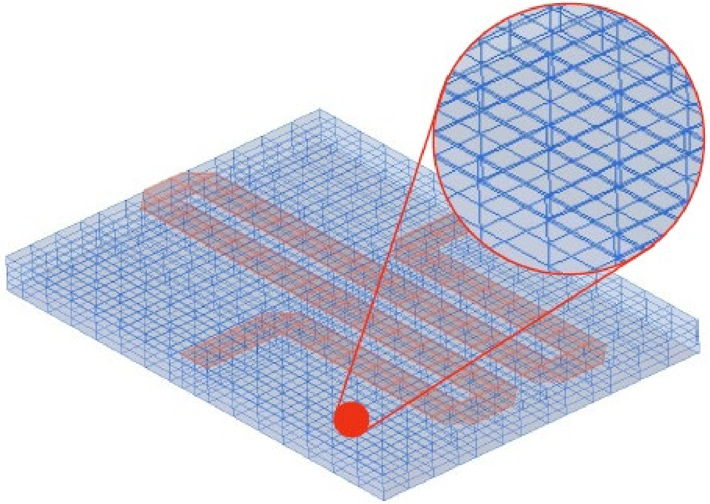
Method	Advantages	Disadvantages	Examples
FEM	Conformal meshes model curved surfaces well. Handles dispersive materials. Good for finding eigenmodes. Can be linked to other FEM solvers (thermal, mechanical, etc.)	Does not handle nonlinear materials easily. Meshes can get very large and limit the computation. Solvers are often proprietary.	High Frequency Structure Simulator (HFSS) and other frequency-domain solvers, including the COMSOL RF module.
FDTD	Handles nonlinearity and wideband signals well. Less limited by mesh size than FEM – better for electrically-large structures. Easy to parallelize and solve with GPUs.	Not good for high-Q devices or dispersive materials. Staircased grid does not model curved surfaces well.	CST Microwave Studio, XFDTD

Example CEM Mesh and Grid



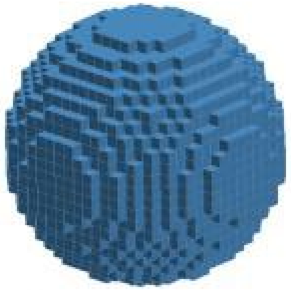


Examples FEM triangular / tetrahedral meshes



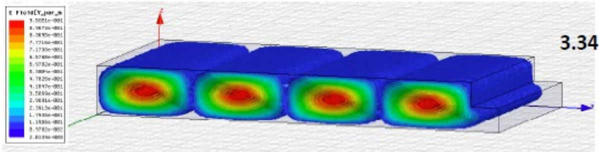
Dave Morris, Agilent 5990-9759

Example FDTD grid with 'Yee' cells



CEM Solvers

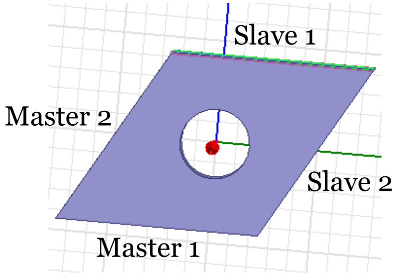
Eigenmode: Closed system, finds the eigen-frequencies and Q values

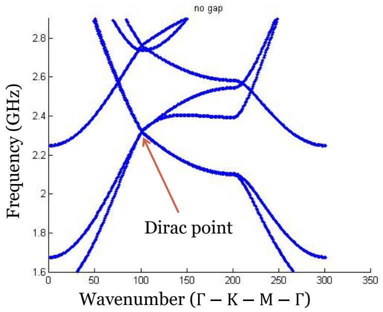


3.346559068 GHz (mode 40)

Harita Tenneti, UMD

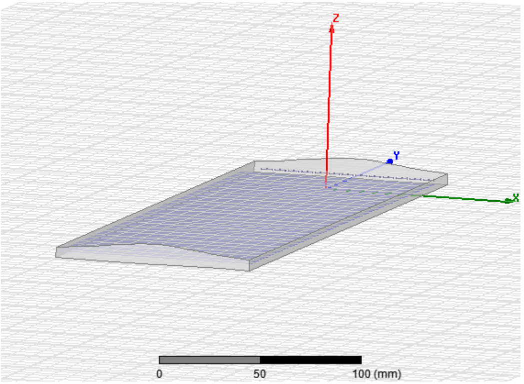
Bo Xiao, UMD



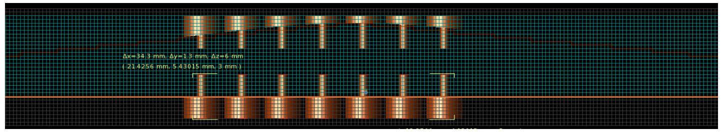


Driven: The system has one or more ‘ports’ connected to infinity by a transmission line or free-space propagating mode. Calculate the Scattering (S) Parameters.

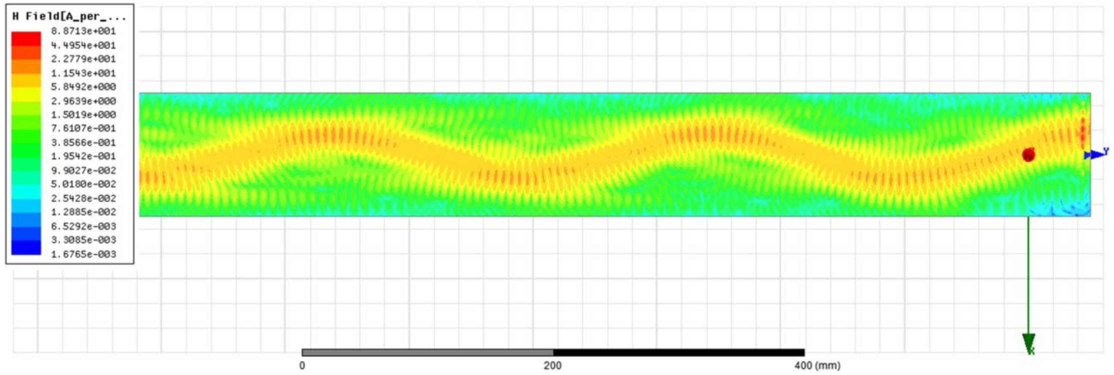
Driven anharmonic billiard



Rahul Gogna, UMD

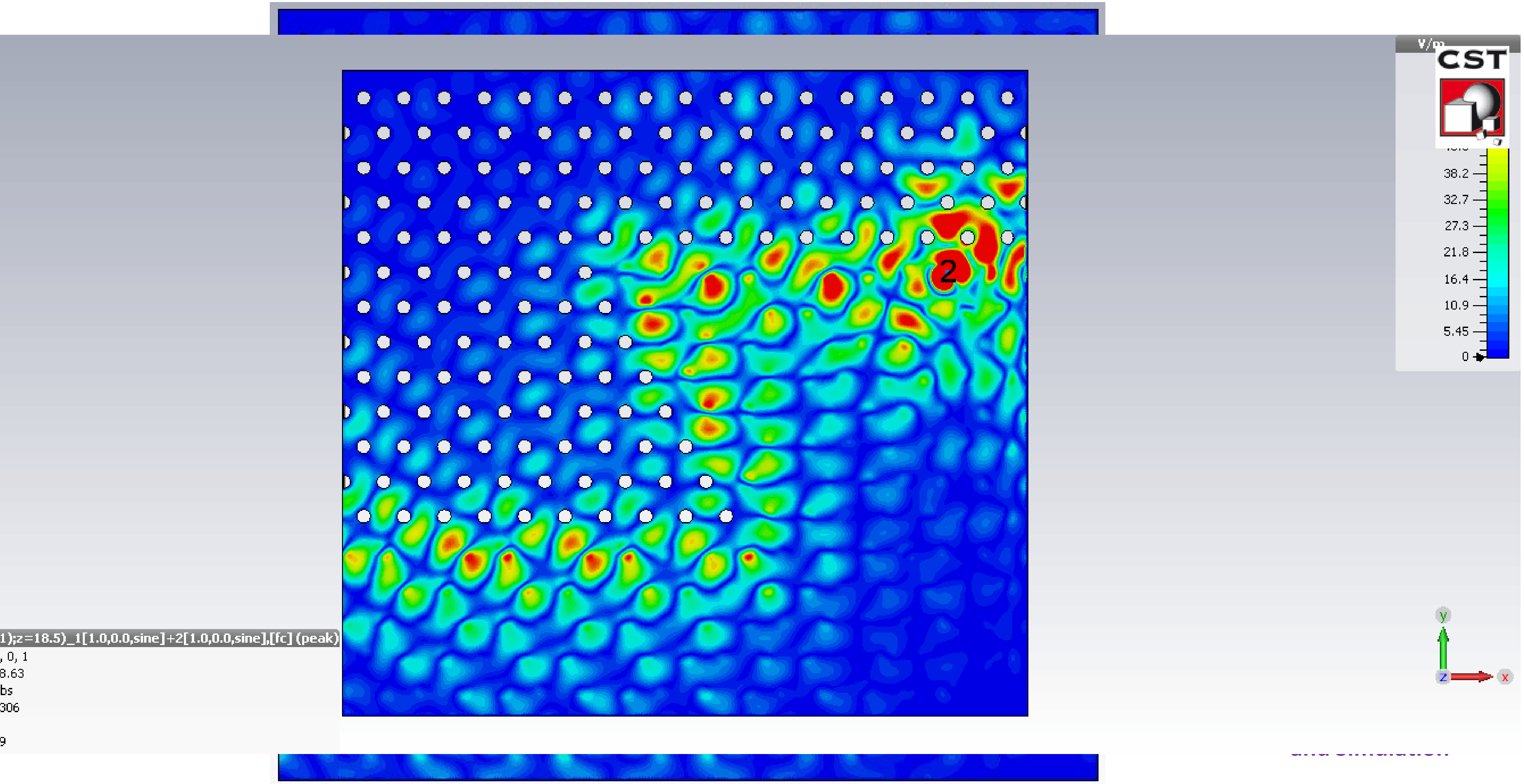


Gaussian wave-packet excitation with antenna array



CEM Solvers (Continued)

Transient (FDTD): The system has one or more ‘ports’ connected to infinity by a transmission line or free-space propagating mode. Calculate the transient signals.



Computational Electromagnetics

- Uses
    - Finding unwanted modes or parasitic channels through a structure
    - Understanding and optimizing coupling
    - Evaluating and minimizing radiation losses in CPW and microstrip
    - Current + field profiles / distributions