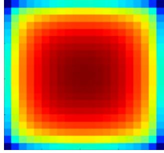
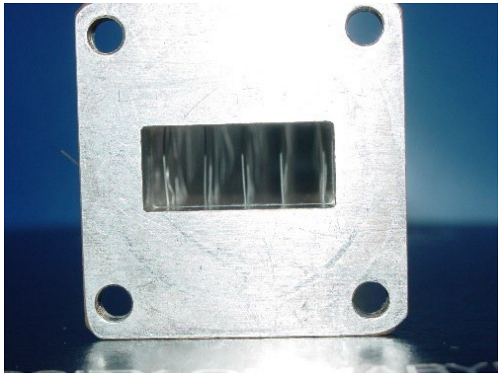
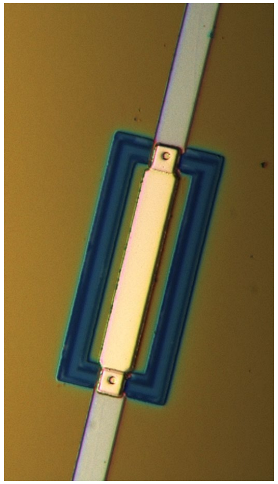
Superconducting Inductance



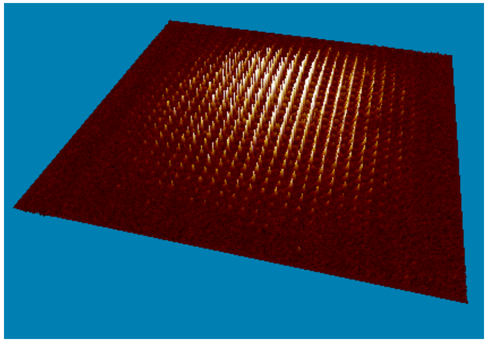
Steven M. Anlage

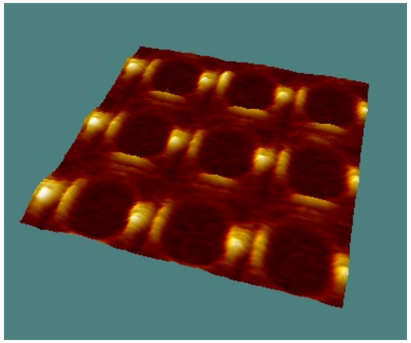




Quantum Materials Center

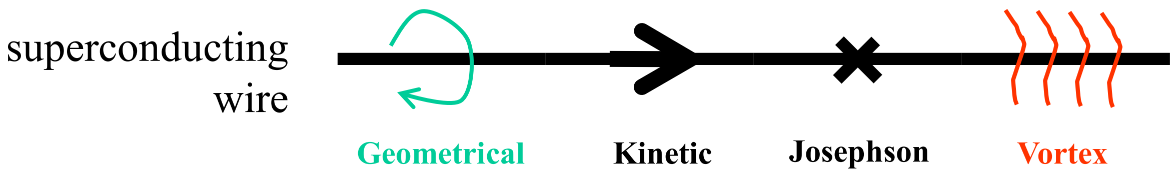
DEPARTMENT OF PHYSICS
UNIVERSITY OF MARYLAND





Research supported by the U.S. Department of Energy, Office of High Energy Physics,
under Award DE-SC0017931,
and NSF/DMR 2004386

Non-Mechanical Tunability of Superconducting Wires



Geometrical Inductance, L_{geo}

Kinetic Inductance, $L_{Kinetic}$

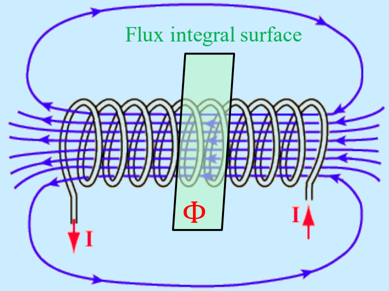
Josephson Inductance, L_{JJ}

Vortex Inductance, L_{vortex}

Kinetic Inductance

$$L = L_{geo} + L_{kinetic}$$

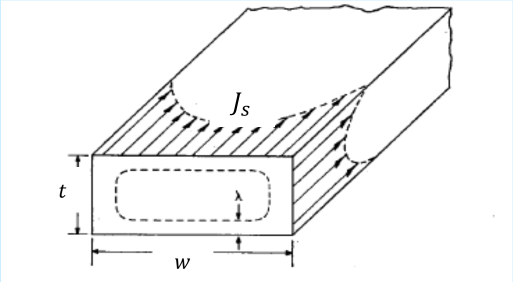
$$L_{geo} = \Phi / I$$



A measure of energy stored in magnetic fields both outside and inside the conductor

$$U_{mag} = \frac{1}{2} L_{geo} I^2$$

$$L_{kinetic} \equiv \frac{\mu_0}{I^2} \iiint \lambda^2(x,y,z)\, J_s^2(x,y,z)\, dV$$



$$U_{kinetic} = \frac{1}{2} L_{kinetic} I^2$$

A measure of energy stored in dissipation-less currents inside the superconductor

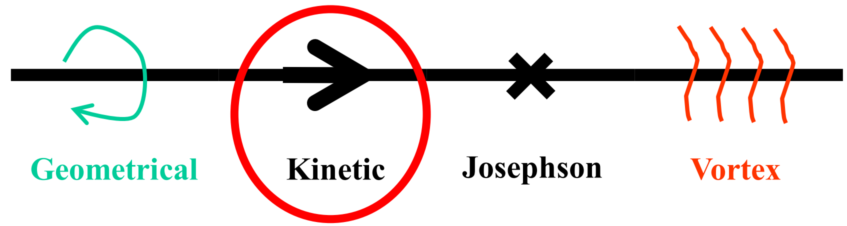
For a current-carrying strip conductor: $L_{kinetic} / \ell \cong \frac{\mu_0 \lambda}{w} \coth(\frac{t}{\lambda})$ (valid when $\lambda << w$, see Orlando+Delin)

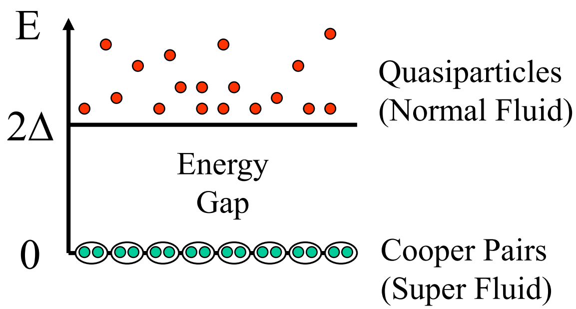
In the limit of $t << \lambda$ or $w << \lambda$: $L_{kinetic} / \ell \propto \frac{\mu_0 \lambda^2}{t}$

… and this can get very large for low-carrier density metals (e.g. TiN or $Mo_{1-x}Ge_x$), or near T_c

Enhanced Kinetic Inductance in Thin Wires

superconducting wire





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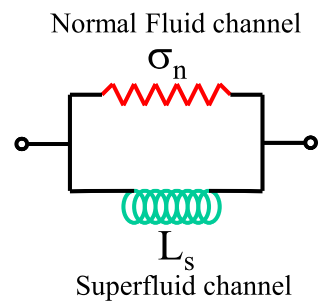
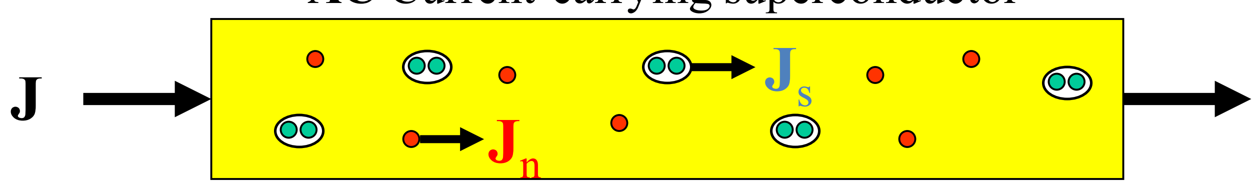
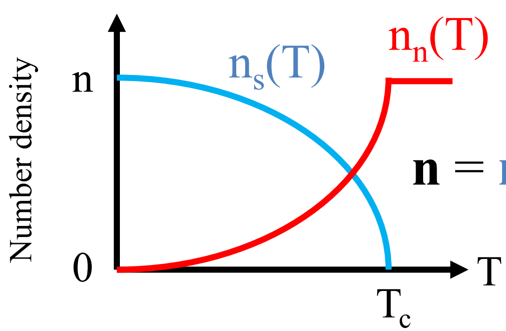


AC Current-carrying superconductor



$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n$



$\mathbf{n} = \mathbf{n_s(T)} + \mathbf{n_n(T)}$

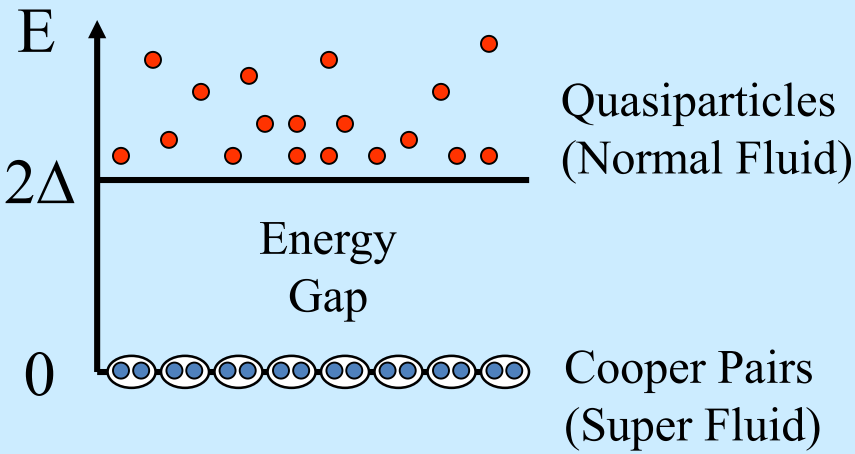
$\mathbf{J}_s = (\text{-e})\, n_s\, \mathbf{v}_s$

As $T \rightarrow T_c$, $n_s \downarrow$, $\mathbf{v}_s \uparrow$, to maintain $\mathbf{J}_s$

$$L_{kinetic} \propto \frac{\lambda^2}{t} \propto \frac{1}{n_s\, t}$$

assumes thickness $t \ll \lambda$

Electrodynamics of Superconductors in the Meissner State (Two-Fluid Model)



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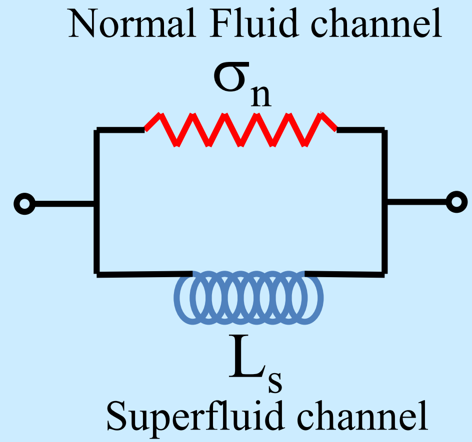
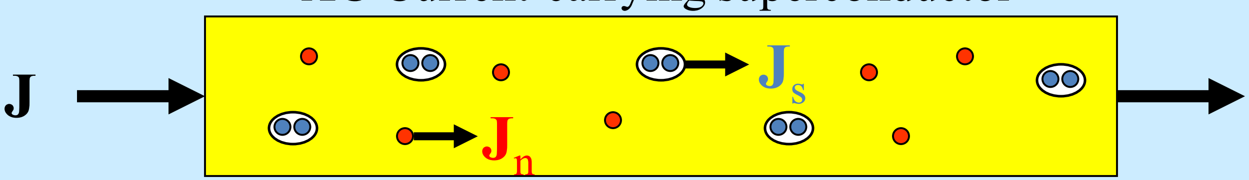


AC Current-carrying superconductor



$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n$

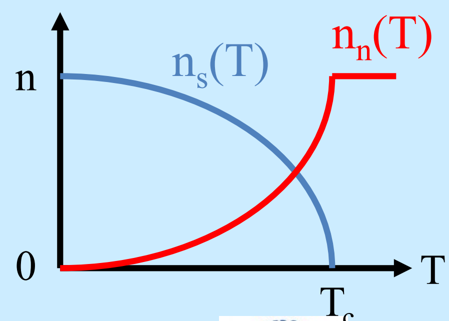
$\mathbf{n} = \mathbf{n}_s(\mathbf{T}) + \mathbf{n}_n(\mathbf{T})$

$J = \sigma E$

$\sigma = \sigma_n - i\,\sigma_2$

$\sigma_n = n_n e^2 \tau / m$

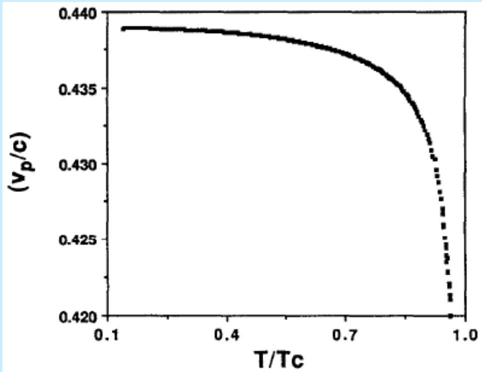
$\sigma_2 = n_s e^2 / m\omega$



n_n = number of QPs
n_s = number of SC electrons
τ = QP momentum relaxation time
m = carrier mass
ω = frequency

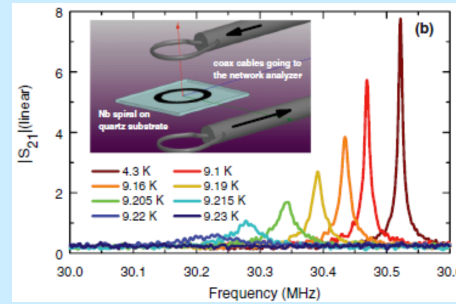
Kinetic Inductance (Continued)

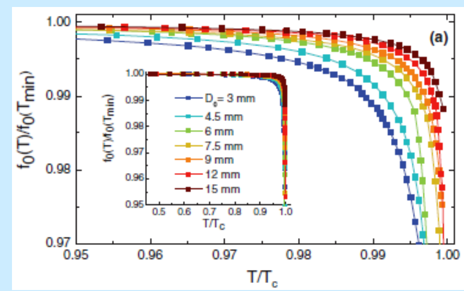
Microstrip HTS transmission line resonator (B. W. Langley, RSI, 1991)



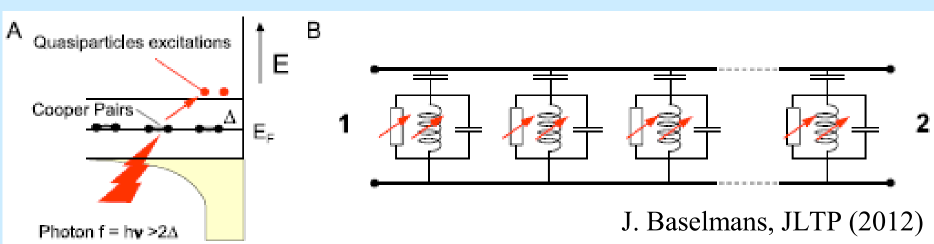
Plasmonic scaling of superconducting metamaterials

(C. Kurter, PRB, 2013)

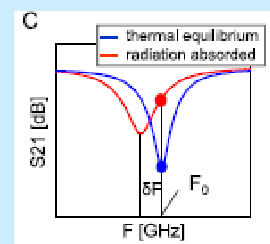




Microwave Kinetic Inductance Detectors (MKIDs)



J. Baselmans, JLTP (2012)



Magnetically-Tuned Kinetic Inductance

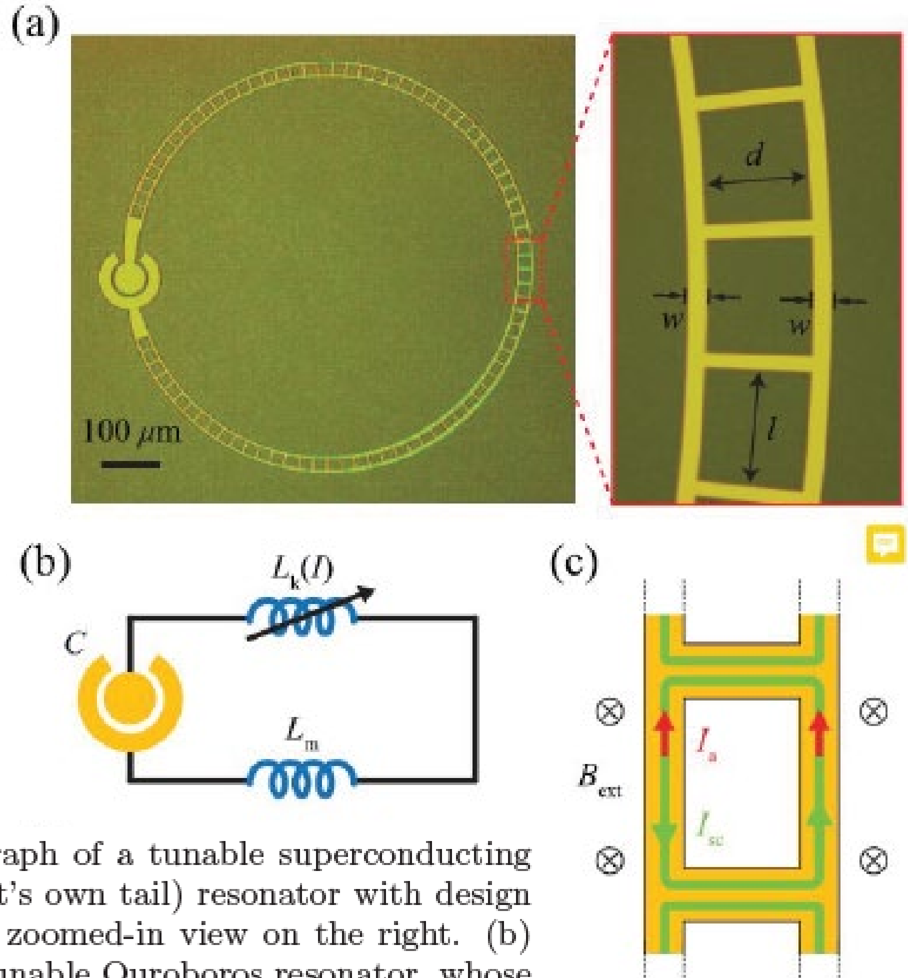
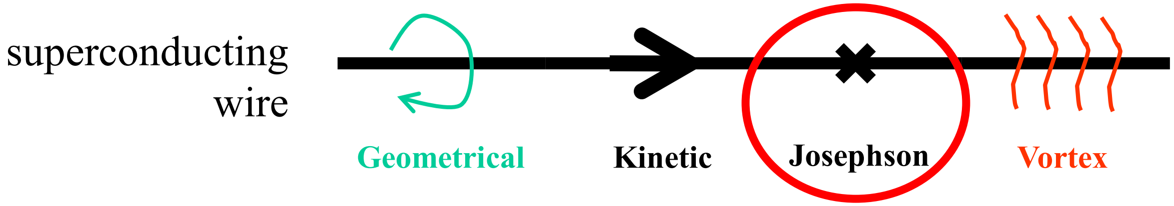


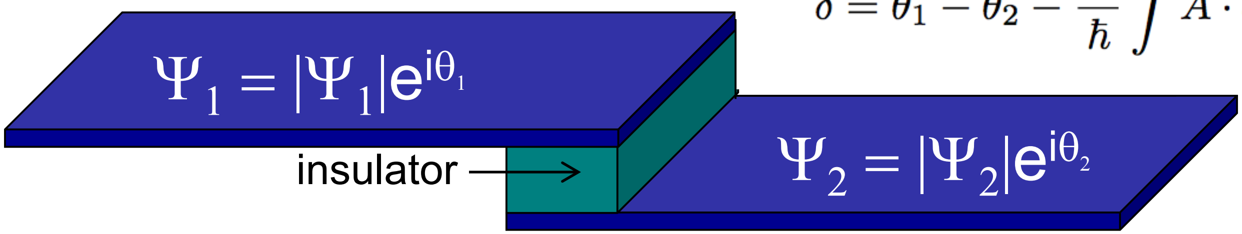
FIG. 4. Optical micrograph of a tunable superconducting Ouroboros (snake eating it's own tail) resonator with design parameters labeled in the zoomed-in view on the right. (b) Equivalent circuit of the tunable Ouroboros resonator, whose inductance is constituted of both geometric inductance L_m and kinetic inductance $L_k(I)$. (c) Frequency tuning mechanism by kinetic inductance. I_{sc}: DC screening currents induced by the external magnetic field. I_a: AC current of the resonant mode.

M. Xu, X. Han, W. Fu, C.-L. Zou, and H. X. Tang, Frequency-tunable high-q superconducting resonators via wireless control of nonlinear kinetic inductance, Applied Physics Letters **114**, 192601 (2019).

Enhanced Josephson Inductance in Thin Wires

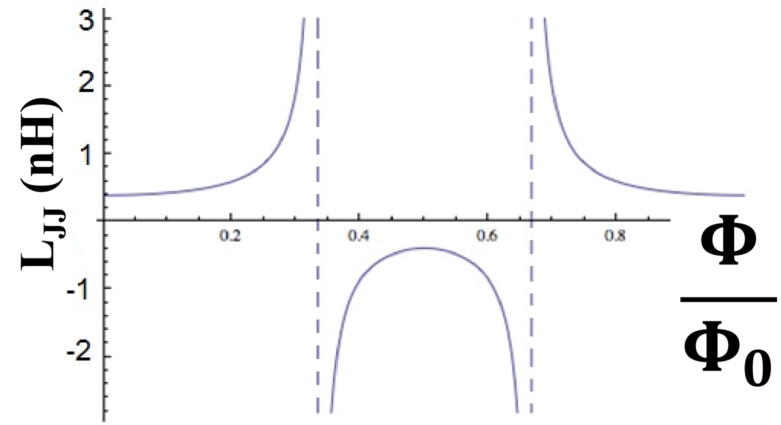


$$\delta = \theta_1 - \theta_2 - \frac{2e}{\hbar}\int \vec{A}\cdot d\vec{l}$$

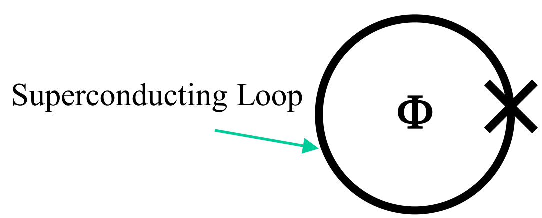


$$\left.\begin{aligned} I &= I_C \sin\delta \\ \frac{d\delta}{dt} &= \frac{2eV}{\hbar} \end{aligned}\right\} \quad L_{JJ} = \frac{\Phi_0}{2\pi I_C \cos\delta}$$

$$\Phi_0 = \frac{h}{2e}$$

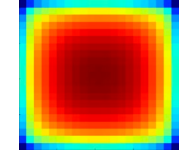


One way to control δ: embed the JJ in a loop

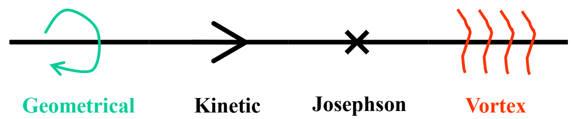


$$\delta \rightarrow \frac{\Phi}{\Phi_0}$$

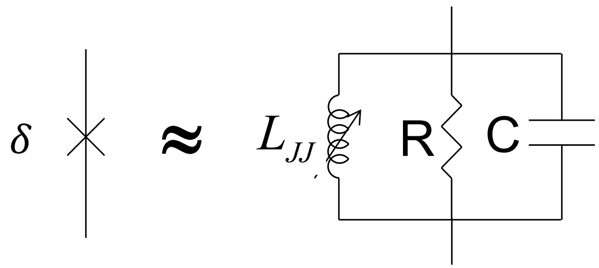
Magnetic flux acts as a surrogate for δ



Josephson Inductance



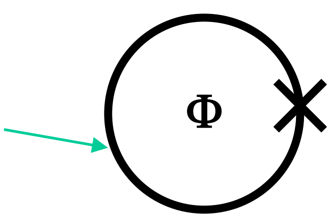
Josephson Inductance is large, tunable and nonlinear



Resistively and Capacitively Shunted Junction (RCSJ) Model

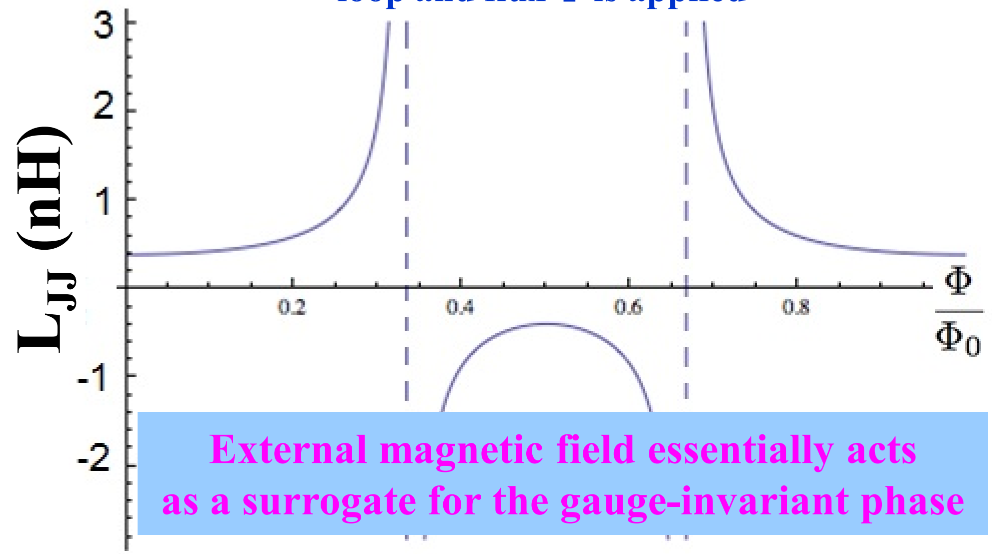
$$L_{JJ} = \frac{\Phi_0}{2\pi I_c \cos(\delta)}$$

The "third Josephson effect"



Combines the Josephson effects with flux quantization

When the JJ is incorporated into a loop and flux Φ is applied

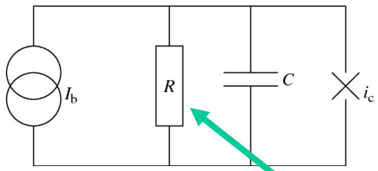


Tuning Josephson Inductance

DC current tuning

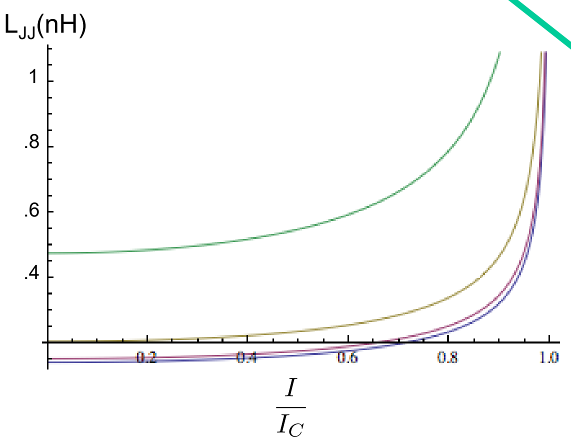
$$I = I_c \sin\delta$$

Ideal dc current source

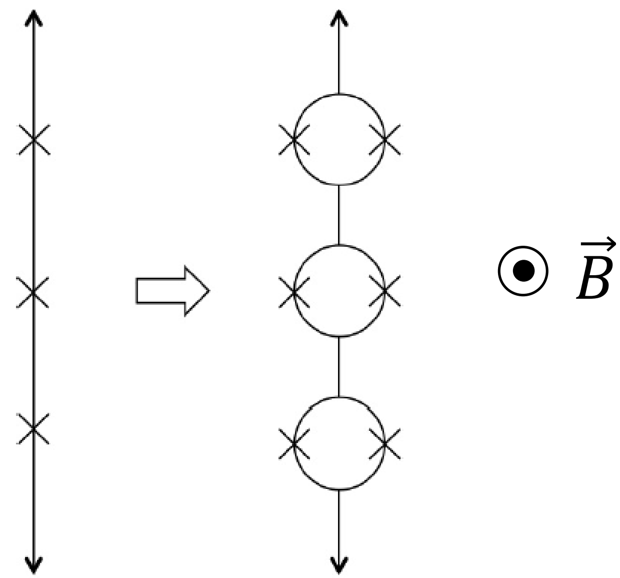


$$\delta = \sin^{-1}\frac{I}{I_c}$$

$$L_{JJ} = \frac{\Phi_0}{2\pi I_c \cos\delta} = \frac{\Phi_0}{2\pi I_c(T)\sqrt{1-(I/I_c)^2}}$$



Magnetic field tuning

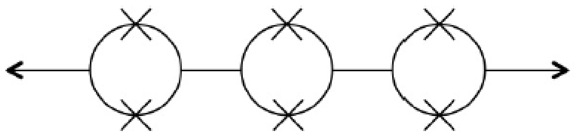


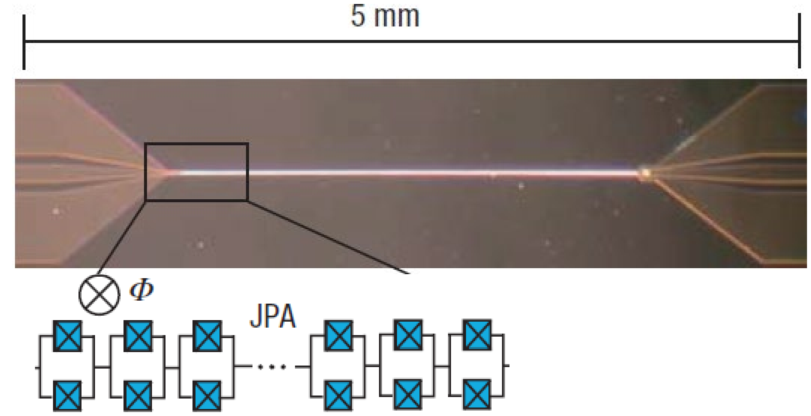
DC SQUID arrays

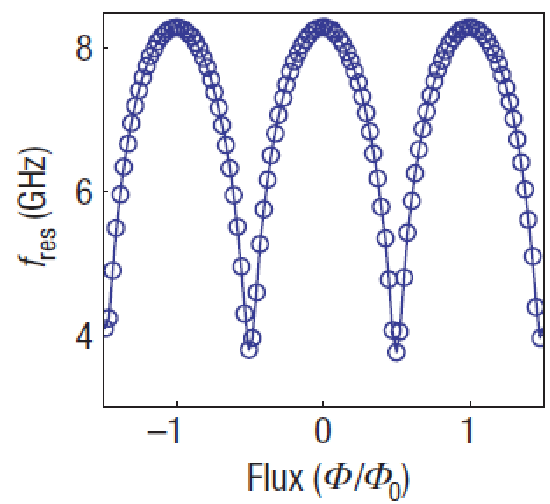
Losses arise from quasiparticle tunneling through the barrier

J. C. Gallop, Phil. Trans. A, **363,** 2221-2247 (2005).

Flux-Tuned DC SQUID-Decorated Wire

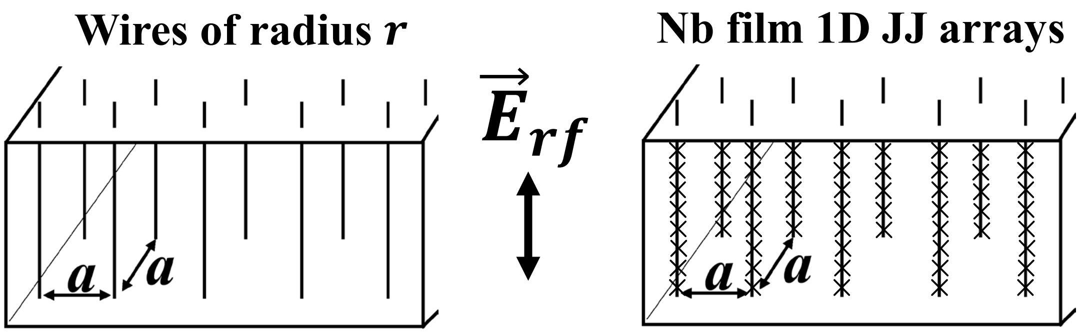


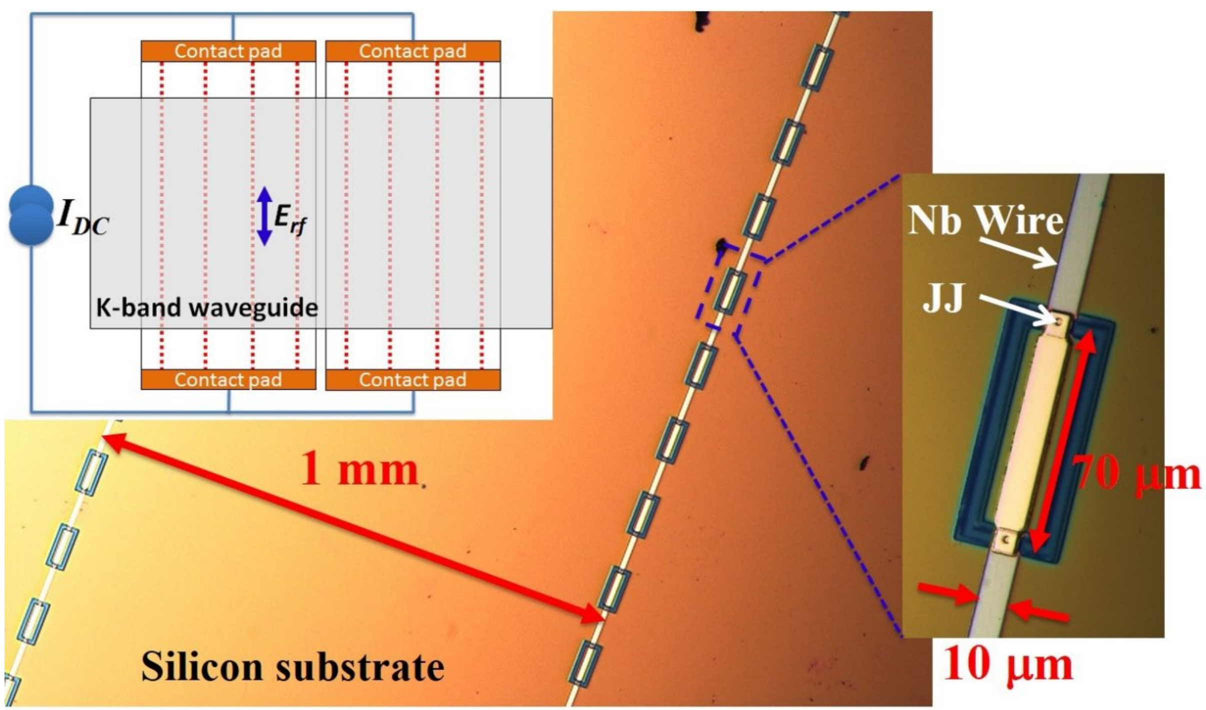




M. A. Castellanos-Beltran, K. D. Irwin, G. C. Hilton, L. R. Vale, and K. W. Lehnert, Amplification and squeezing of quantum noise with a tunable josephson metamaterial, Nature Physics 4, 928 (2008).

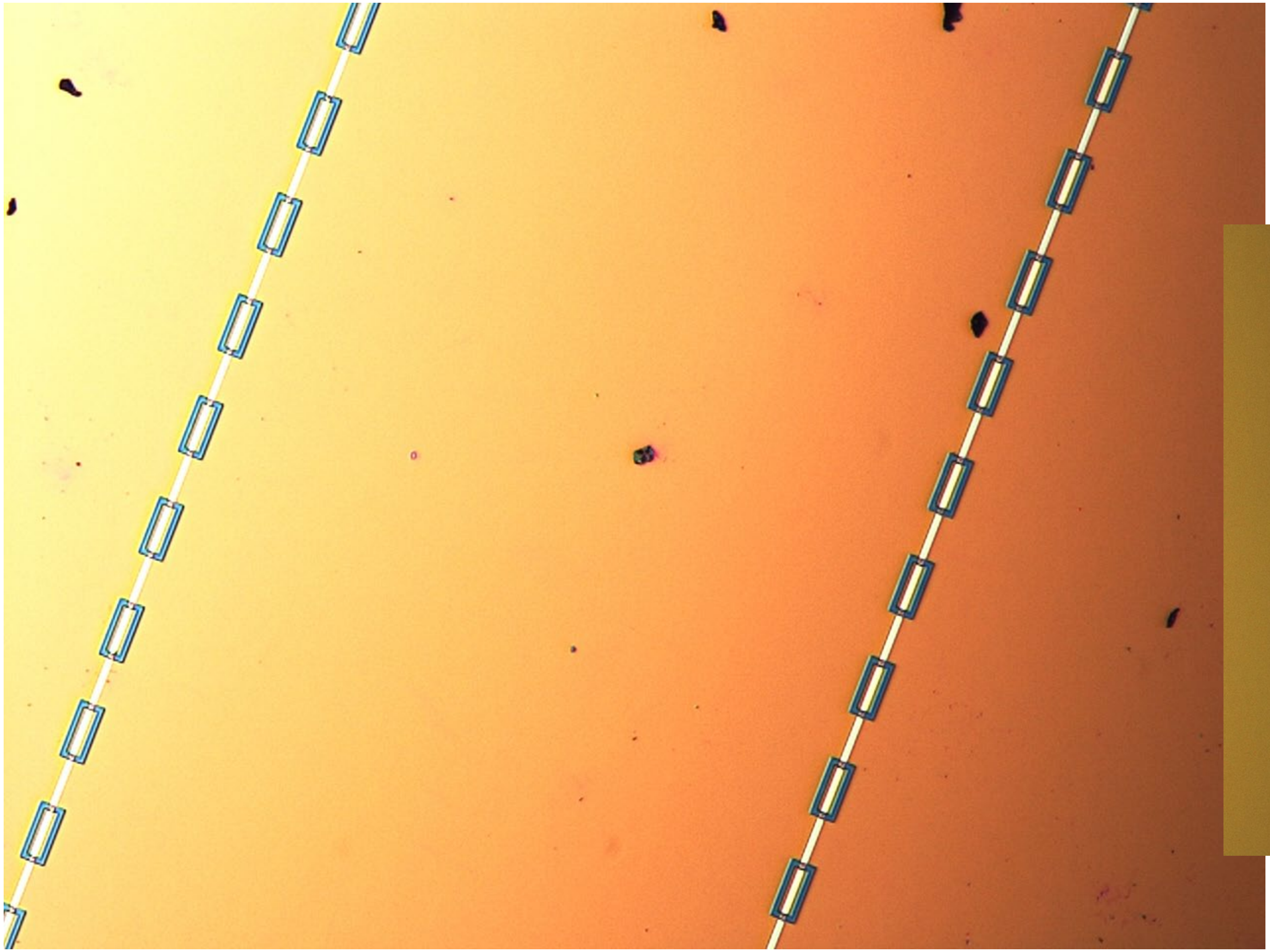
Josephson Junction Decorated Niobium Wires

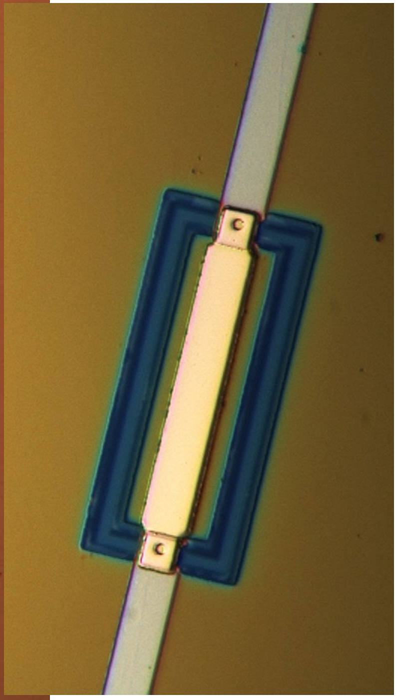




Melissa Trepanier, Daimeng Zhang, Lyudmila Filippenko, Valery Koshelets, Steven M. Anlage, "**Tunable Superconducting Josephson Dielectric Metamaterial**," AIP Advances 9, 105320 (2019)

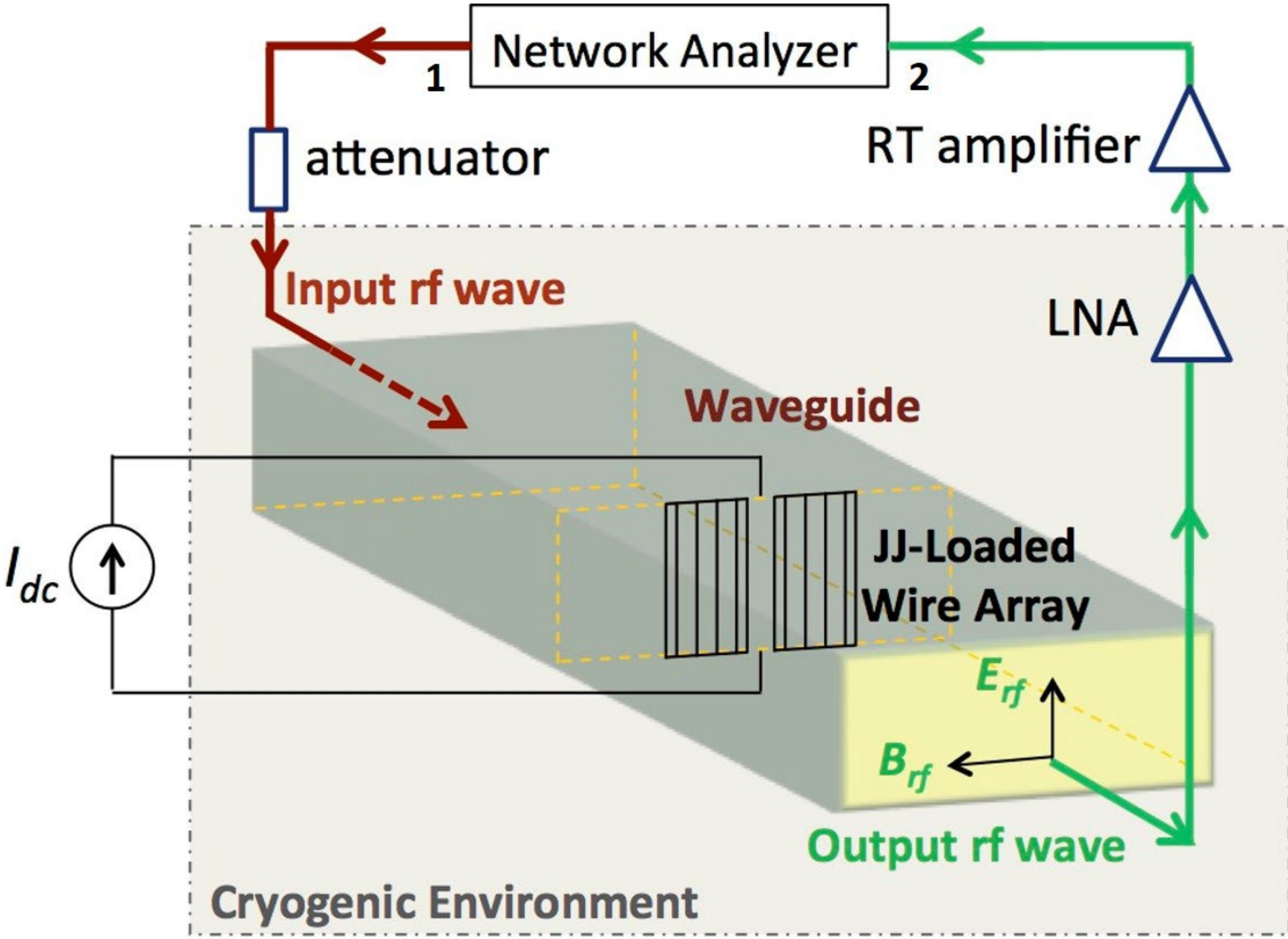
JJ-Decorated Wire Arrays





Experimental Setup

Transmission Through Superconducting Metamaterials



Measure transmission, S_{21} as a function of frequency

Thin and Narrow Superconducting Wires

Which Are Best for *Low-Loss* Tunability?

There are two basic kinds of materials that achieve high kinetic inductance

Disordered materials (e.g. NbN_x, amorphous Mo-Ge alloys)

Granular materials (small grains separated by networks of Josephson junctions)

e.g. GrAl

GrAl – can be made simply and patterned into thin narrow lines

Al has a low melting point - compatible with liftoff lithography

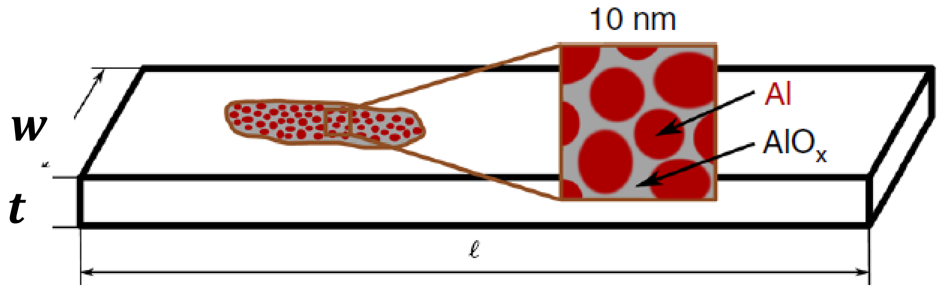
It has been argued [Appl Phys Lett 117, 062601 (2020)] **that:**

Disordered superconductors have many "mid-gap states" and are more lossy

Granular materials are close to the Mott limit and have fewer mid-gap states

→ Large tunnel barrier resistance *R*

Ideal materials: films made up of small (few nm) size grains with narrow size distribution and Josephson coupling between them

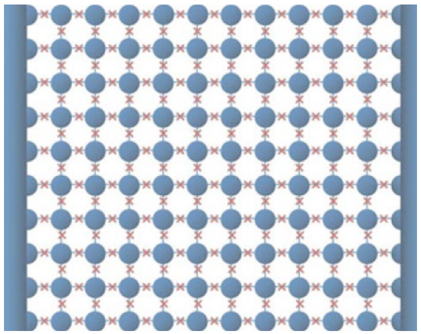


$$\frac{\lambda^2}{t} > 8\ \mu m$$

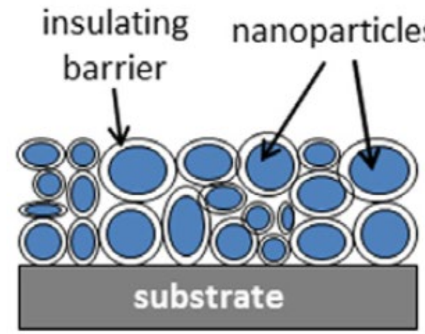
$t = 20\ nm$

N. Maleeva, L. Grünhaupt, T. Klein, F. Levy-Bertrand, O. Dupre, M. Calvo, F. Valenti, P. Winkel, F. Friedrich, W. Wernsdorfer, A. V. Ustinov, H. Rotzinger, A. Monfardini, M. V. Fistul, and I. M. Pop, "Circuit quantum electrodynamics of granular aluminum resonators," Nat. Commun. **9 (1), 3889 (2018).**

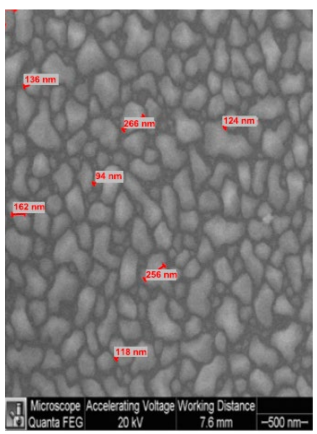
Granular Superconductors



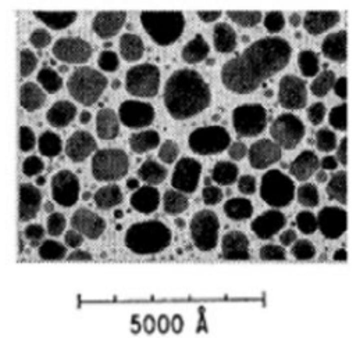
Anna Carbone, Marco Gilli, Piero Mazzetti, and Linda Ponta, "Array of Josephson junctions with a nonsinusoidal current-phase relation as a model of the resistive transition of unconventional superconductors," J Appl Phys **108 (12), 123916 (2010).**



GRANULAR THIN FILMS



Bar Hen, Xinyang Zhang, Victor Shelukhin, Aharon Kapitulnik, and Alexander Palevski, "Superconductor–insulator transition in two-dimensional indium–indium-oxide composite," Proceedings of the National Academy of Sciences **118 (2), e2015970118 (2021).**



EM OF GRANULAR THIN FILM OF Sn ON OXIDIZED Al

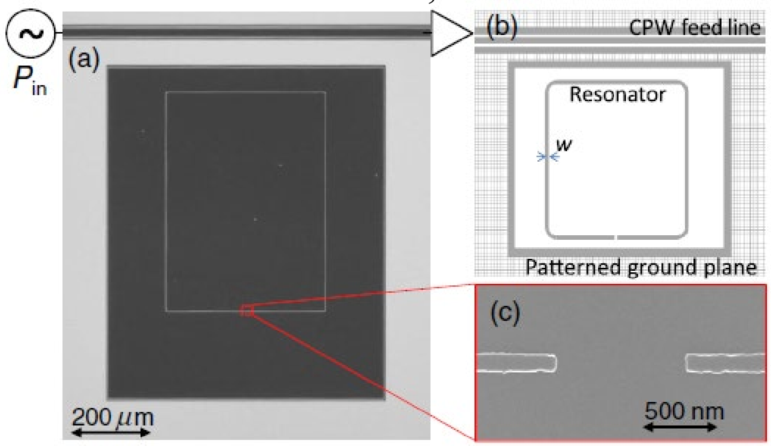
Sangita Bose and Pushan Ayyub, "A review of finite size effects in quasi-zero dimensional superconductors," Rep Prog Phys **77, 116503 (2014).**

One issue with granular thin film superconductors: uniformity and homogeneity

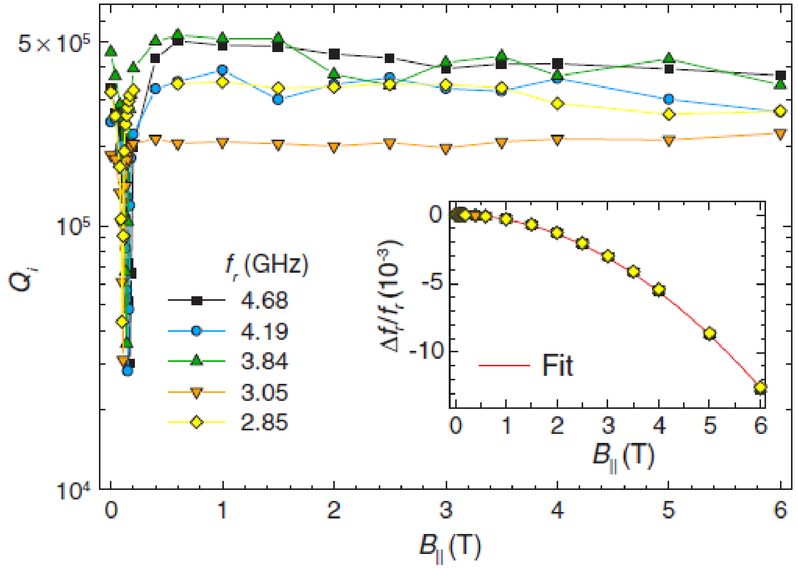
Thin Superconducting Film in a Parallel Magnetic Field

Thin film microwave resonator operating up to $B_{||}$ = 6 T

NbTiN nanowire SRR resonator
2.8 – 4.7 GHz, 280 mK

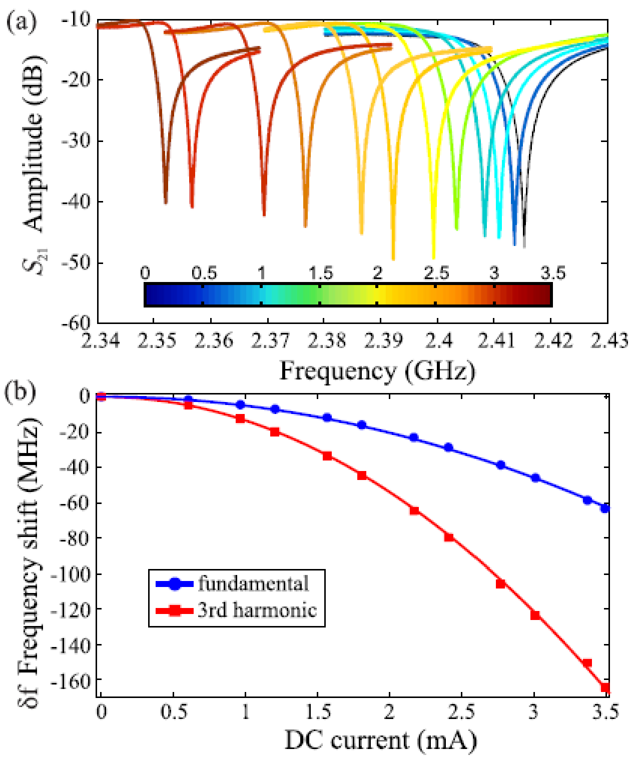


w = 100 nm
t = 8 nm



N. Samkharadze, *et al.*, Physical Review Applied **5** (4), 044004 (2016)

Current-Tunable Kinetic Inductance in Superconducting Wires



$$L_{kinetic}(I_{DC}) \approx L_{kinetic}(0)\left[1+\left(\frac{I_{DC}}{I_*}\right)^2\right]$$

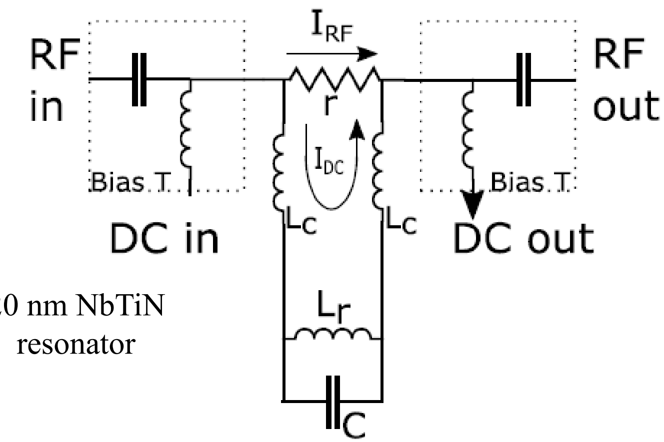
Current-induced depairing: $n_s(I_{DC})$

Microstrip resonator
DC current resonant frequency tuning
50 nm thick NbN
Figure of merit:

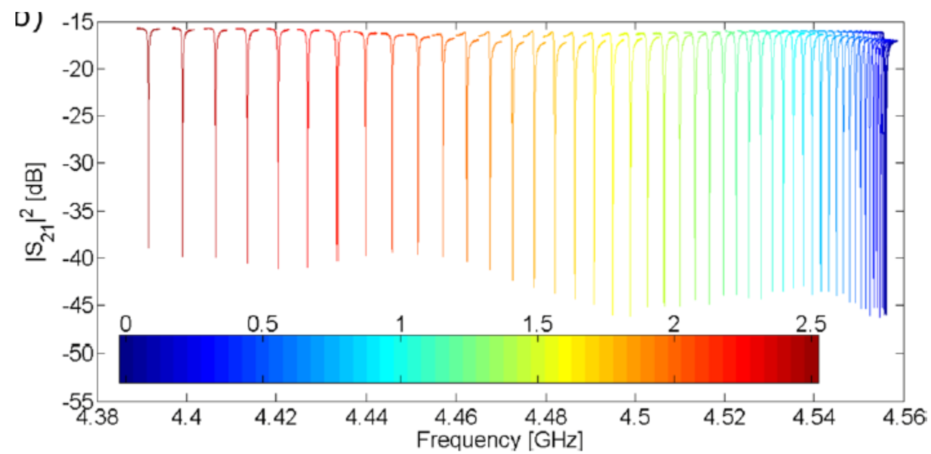
$$Q_i\frac{\delta f(I_{DC})}{f_0} = 150 \text{ at } 2.4 \text{ GHz}$$

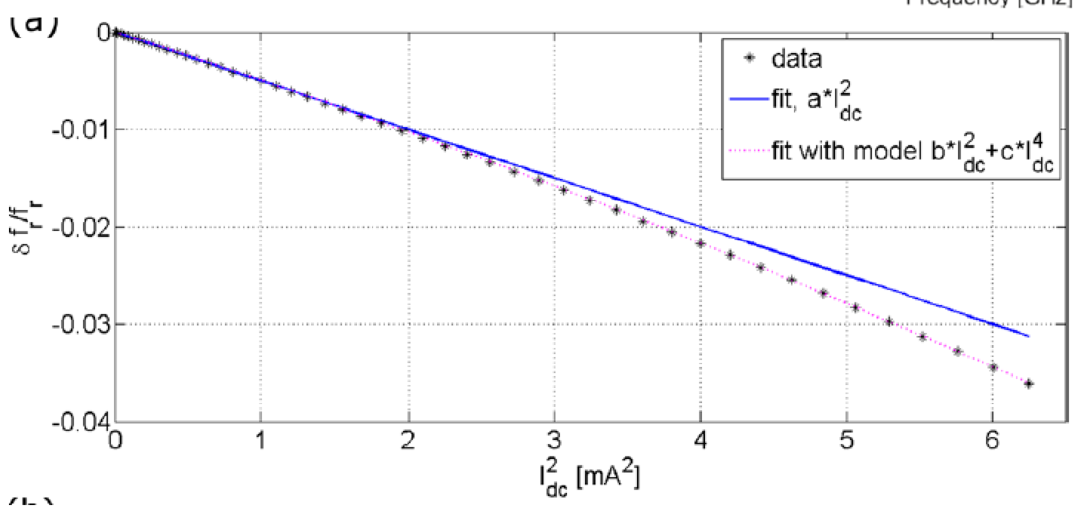
A. A. Adamyan, S. E. Kubatkin, and A. V. Danilov, "Tunable superconducting microstrip resonators," Appl Phys Lett **108** (17), 172601 (2016).

Current-Tunable Kinetic Inductance in Superconducting Wires



20 nm NbTiN resonator

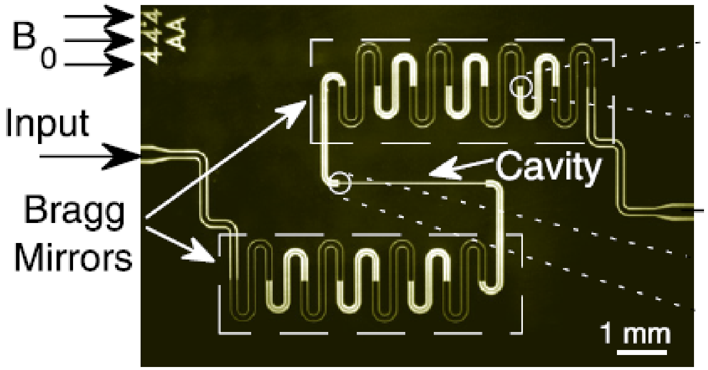


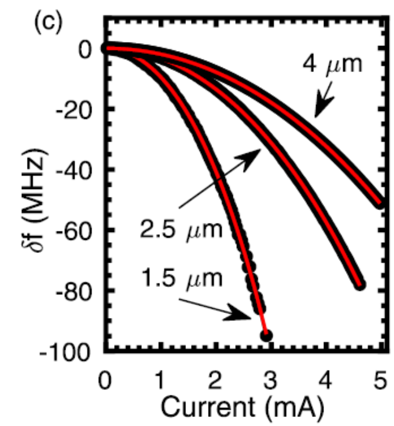


M. R. Vissers, J. Hubmayr, M. Sandberg, S. Chaudhuri, C. Bockstiegel, and J. Gao, "Frequency-tunable superconducting resonators via nonlinear kinetic inductance," Appl Phys Lett **107** (6), 062601 (2015)

Current-Tunable Kinetic Inductance in Superconducting Wires

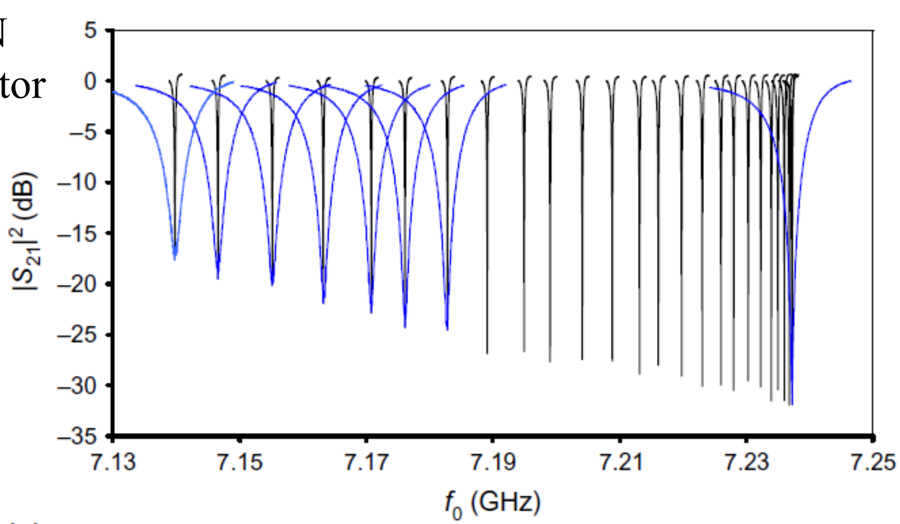
20 nm NbTiN
resonator
B_0 = 275 mT

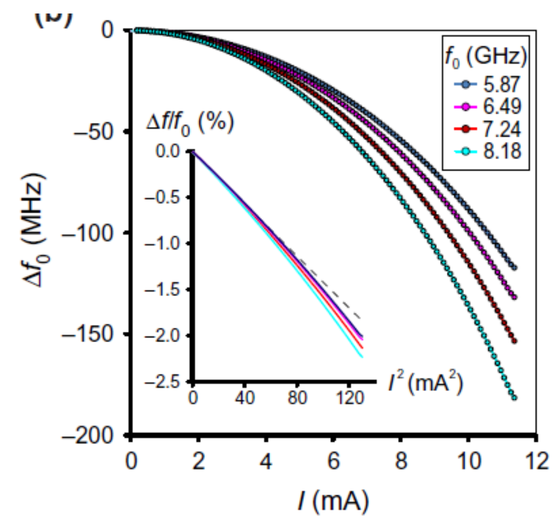




A. T. Asfaw, *et al.*, Appl Phys Lett **111** (3), 032601 (2017)

NbN
resonator





S. Mahashabde, *et al.*, Phys. Rev. Applied 14, 044040 (2020)

Microwave Losses / Flux Motion

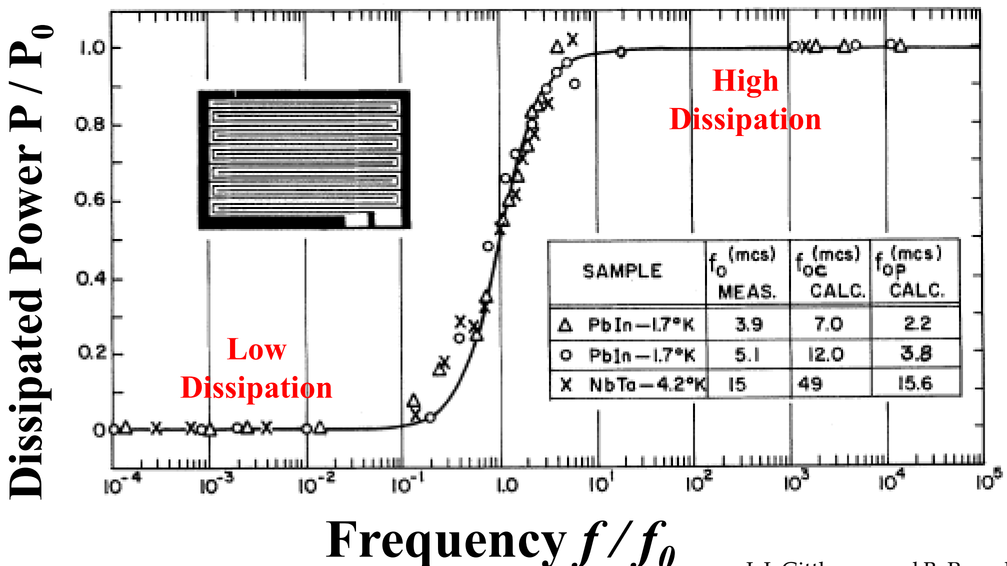
Single-vortex response to AC current (Gittleman-Rosenblum model)

$$m\ddot{\vec{x}} + \eta\dot{\vec{x}} + \vec{F}_{pin} = \vec{J} \times \overrightarrow{\Phi}$$

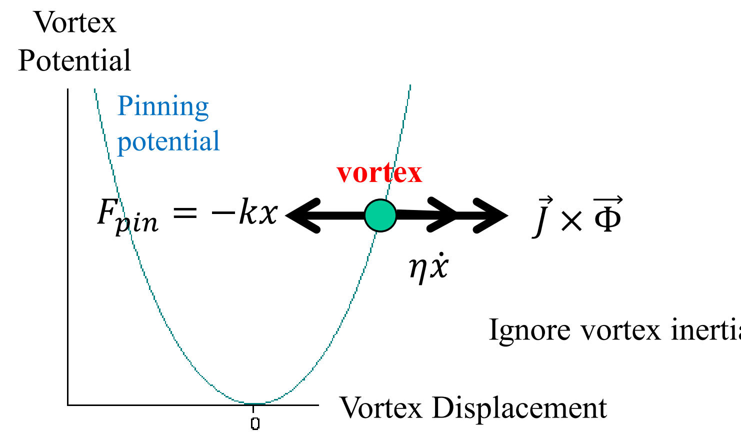
Equation of motion for vortex in a rigid lattice (vortex-vortex force is constant)

m Effective mass of vortex

η Vortex viscosity ~ σ_n



SAMPLE	f_0 (mcs) MEAS.	f_{oc} (mcs) CALC.	f_{op} (mcs) CALC.
△ PbIn—1.7°K	3.9	7.0	2.2
o PbIn—1.7°K	5.1	12.0	3.8
X NbTa—4.2°K	15	49	15.6



Pinning frequency f_0

$$2\pi f_0 \equiv k/\eta$$

J. I. Gittleman and B. Rosenblum, "Radio-frequency resistance in the mixed state for subcritical currents," *Phys. Rev. Lett.*, vol. 16, no. 17, pp. 734–736, 1966.

Experimental range of pinning frequencies: $f_0 \sim 5 - 100\ GHz$ depending on material, temperature, magnetic field

Pinning Frequency of Several Superconductors

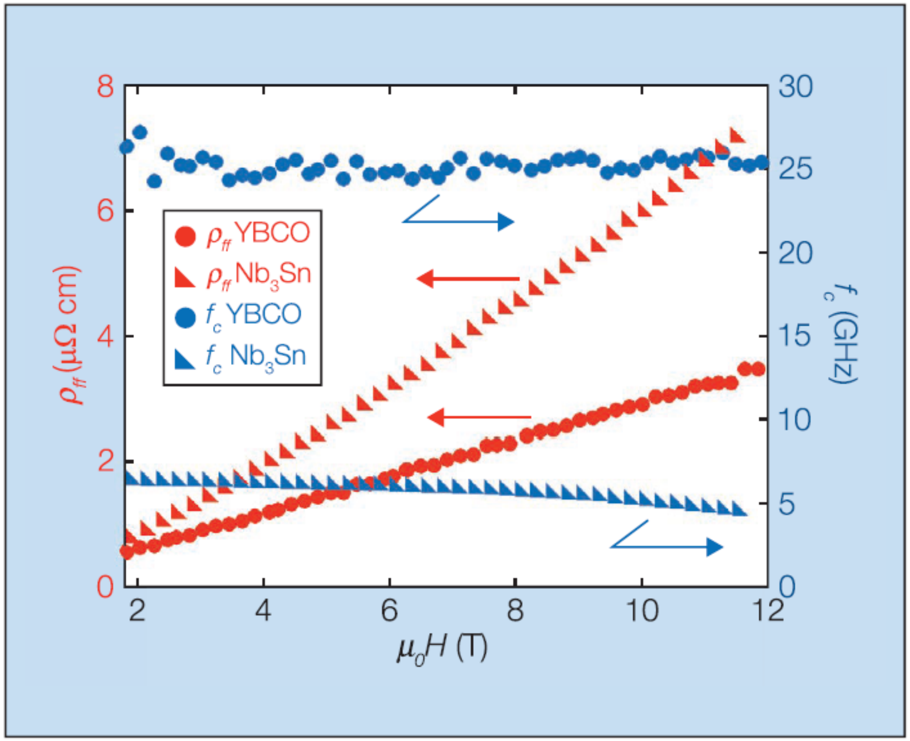


Fig. 4. Flux-flow resistivity (red) and the characteristic frequency (blue) measured at similar reduced temperature T/T_c on a bulk Nb_3Sn sample at 6 K (triangles) and on a YBCO thin film (full dots) at 27 K, for fields up to 12 T.

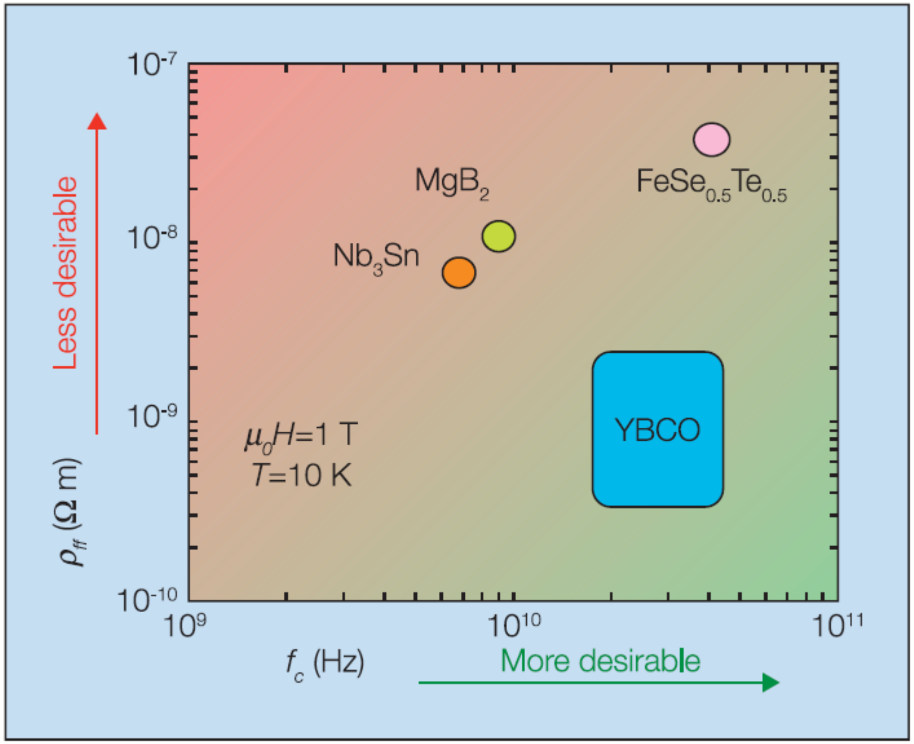
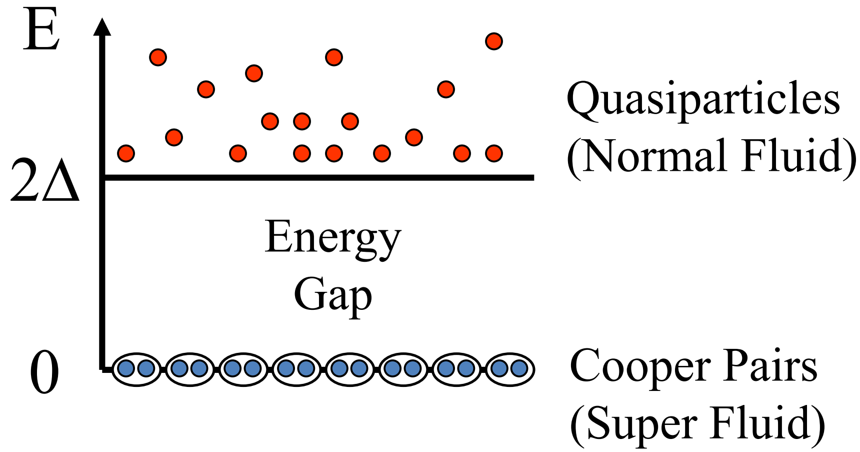


Fig. 5. Plot of the flux-flow resistivity ρ_{ff} and characteristic frequency f_c measured at 10 K and 1 T on different SCs: Nb_3Sn (orange), MgB_2 (green), YBCO (light blue) and $FeSe_{0.5}Te_{0.5}$ (pink).

A. Alimenti, N. Pompeo, K. Torokhtii, and E. Silva, "Surface Impedance Measurements in Superconductors in DC Magnetic Fields: Challenges and Relevance to Particle Physics Experiments," IEEE Instrumentation & Measurement Magazine **24 (9), 12-20 (2021).**

Frequency Limitations of Superconductors



The finite energy gap Δ imposes a frequency limit:

Ideally $\hbar\omega \ll 2\Delta$ to maintain low losses

► $T_c \gg \frac{\hbar\omega}{3.5 k_B} = 1.4\ K$ (for 100 GHz)

(Nb, NbN, NbTiN, MgB_2, Boron-doped diamond, not Al)

In addition, node-less (s-wave) superconductors are desired:

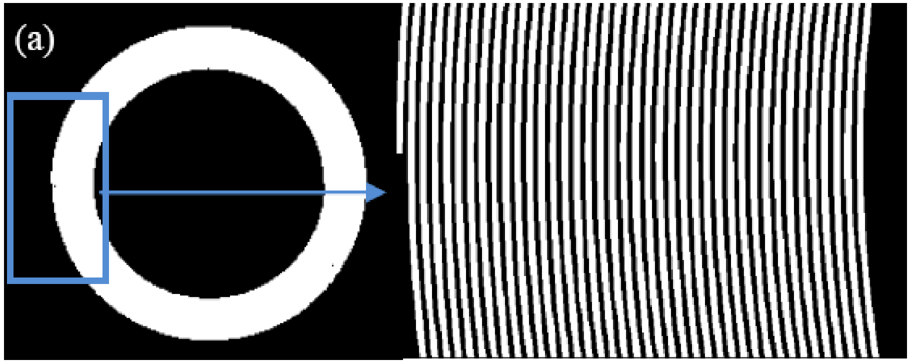
Ensures # Quasiparticles ~ $e^{-\Delta/k_B T} \ll 1$ at low temperatures

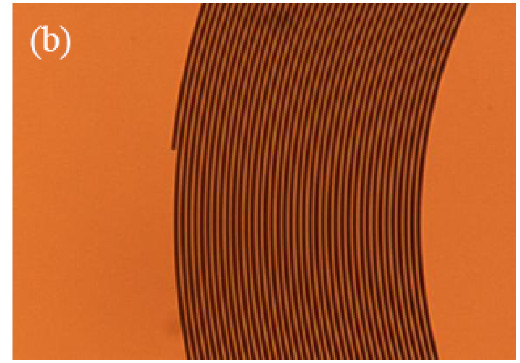
Plasmonic Superconducting RF Metamaterials

Temperature-Dependent Kinetic Inductance

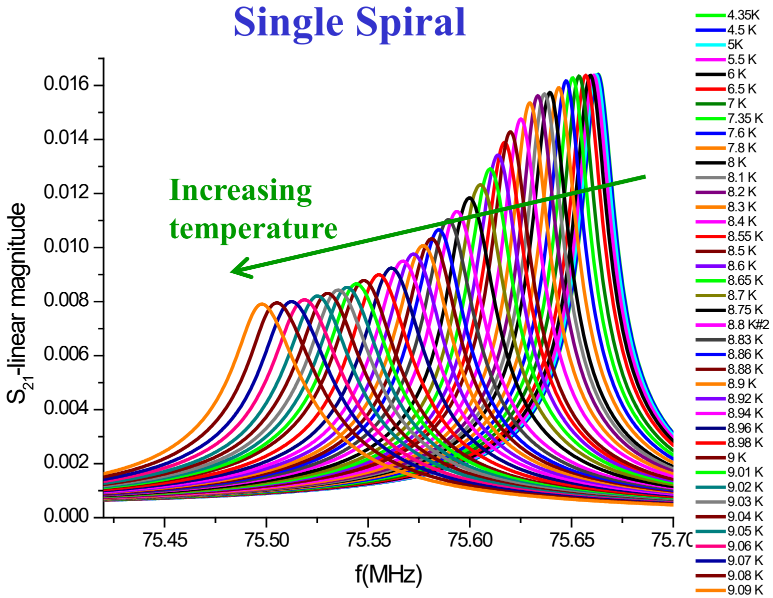
← 6 mm →

10 μm wide x 200 nm thick Nb films

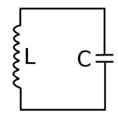




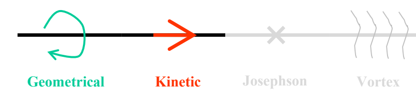
Single Spiral

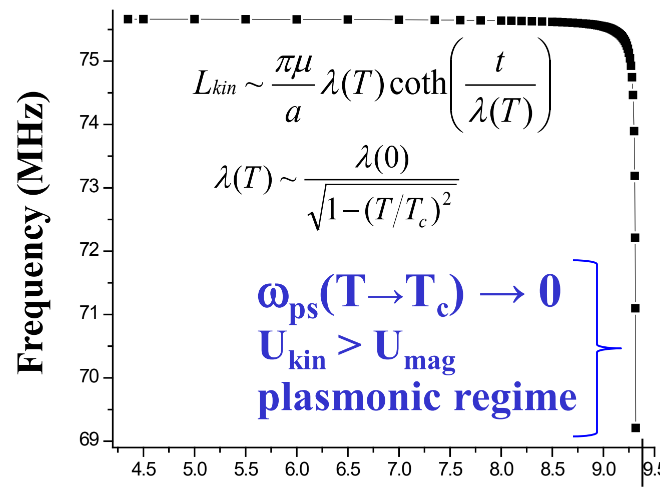


C. Kurter, *et al.*, Appl. Phys. Lett. 96, 253504 (2010)



$$\omega_0 = \frac{1}{\sqrt{LC}}$$





$$L_{kin} \sim \frac{\pi\mu}{a}\lambda(T)\coth\left(\frac{t}{\lambda(T)}\right)$$

$$\lambda(T) \sim \frac{\lambda(0)}{\sqrt{1-(T/T_c)^2}}$$

$\omega_{ps}(T \to T_c) \to 0$

$U_{kin} > U_{mag}$

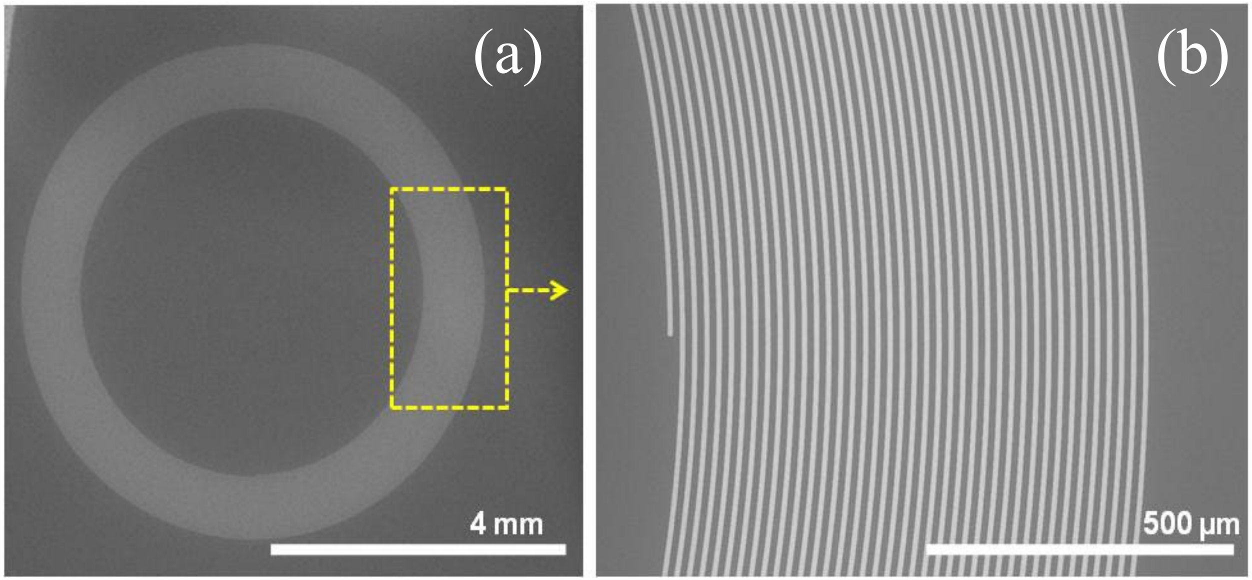
plasmonic regime

T_c

C. Kurter, J. Abrahams, G. Shvets, S. M. Anlage, "**Plasmonic Scaling of Superconducting Metamaterials**," Phys. Rev. B **88**, 180510(R) (2013)

Superconducting Spiral Meta-Atoms

Our Design: Superconducting Thin Film Spiral Resonators



Typical spiral:
200 nm thick.
Made up of lines 10 µm wide, 10 µm spacing, 6 mm OD, 40 turns.

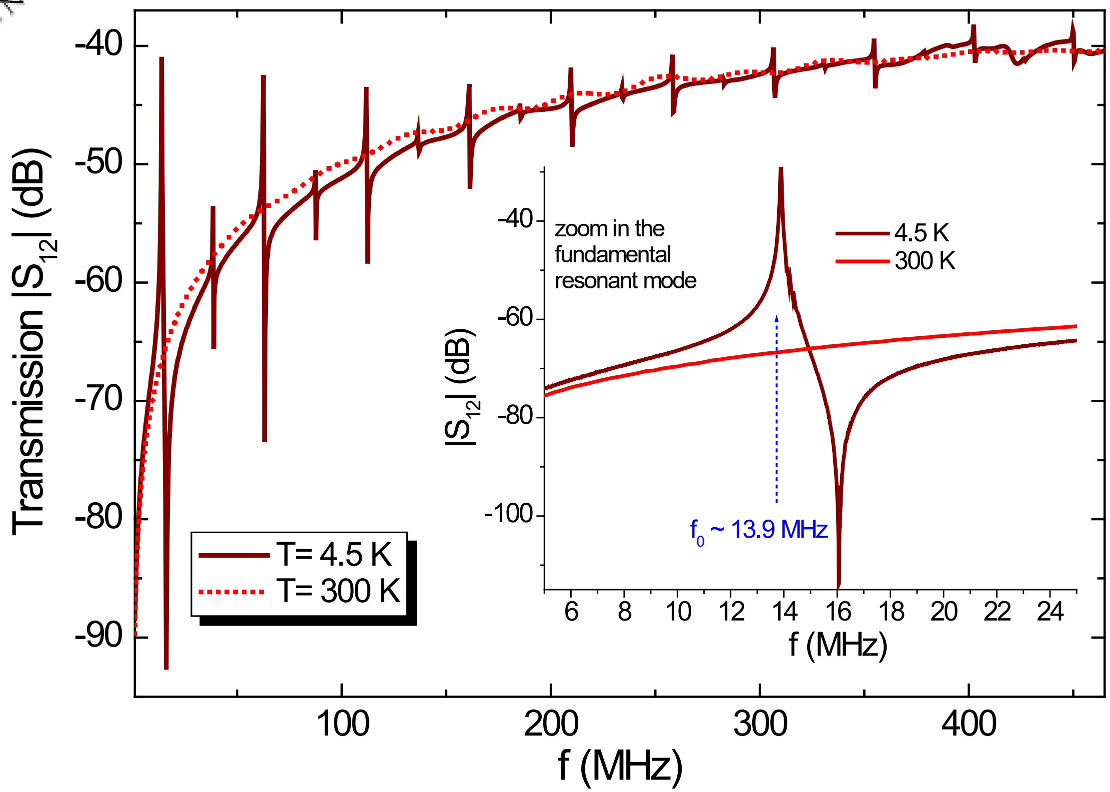
Geometrical and kinetic inductance, along with self-capacitance, make the spirals very compact self-resonant objects – ideal as RF meta-atoms

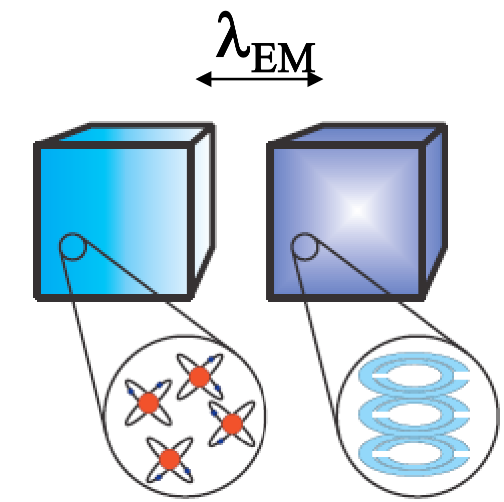
Nb: $T_c = 9.25$ K
$YBa_2Cu_3O_7$: $T_c = 92$ K

C. Kurter, *et al*., Appl. Phys. Lett. 96, 253504 (2010)

A 'Hydrogenic' Low-Frequency Meta-Atom

Dimensions of the f_0=13.5 MHz spirals: D_o=7 mm, N= 200, w=s= 3 μm





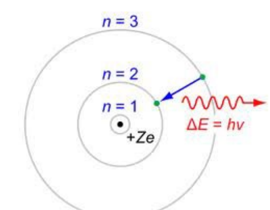
Pendry (2004)

"Q" of a Hydrogen atom ($\lambda/\Delta\lambda$) for the H_α line
'Natural': $Q \sim 14 \times 10^6$
Doppler broadened (sun): $Q \sim 15 \times 10^3$

Loaded Q values as high as 30,000 at 4.5 K

$$\lambda_{EM} \ / \text{atom size} \sim 3{,}000$$

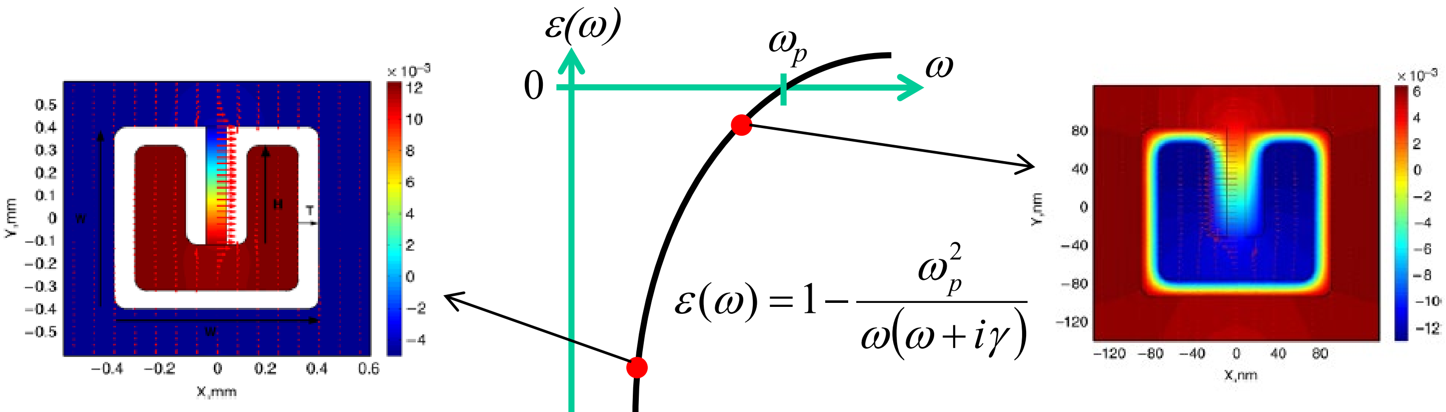
Bohr model for H_α line in the Balmer series for the n=3 to n=2 transition



$$H_\alpha \text{ visible light / Hydrogen atom size} \sim 1{,}000$$

Plasmonic Behavior of Normal Metals
and Artificial Magnetism in the Visible

Perfect Electric Conductor Limit | Plasmonic Limit



$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Strong Magnetic Response
at resonance

$R_{plasma} << 1$

$$\omega_p^2 = \frac{n e^2}{\varepsilon_0 m}$$

Weak Magnetic Response
(small losses destroy $Re[\mu_{eff}] < 0$)

$R_{plasma} >> 1$

Skin depth:
$$\delta \sim \frac{\lambda_{EM}}{\sqrt{-\varepsilon}}$$

A magnetic resonator (SRR, spiral, etc.) stores energy in several forms:

$$U_{total} = U_{magnetic} + U_{electric} + U_{kinetic}$$

The plasmonic parameter is defined as $R_{plasma} = U_{kinetic}/U_{magnetic}$

Loss of magnetic response upon scaling: J. Zhou, *et al.*, PRL (2005)

We are working in the limit of SRR dimension much less than wavelength

Solid State Communications 146 (2008) 208–220

Optical magnetism and negative refraction in plasmonic metamaterials

Yaroslav A. Urzhumov, Gennady Shvets*

Plasmonic Behavior of Superconducting Metamaterials

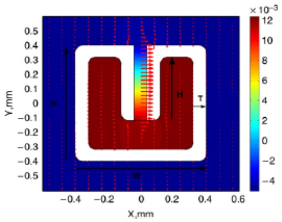
Superconducting Dielectric Function:

$$\varepsilon_{SC}(\omega) = 1 - \frac{\omega_{ps}^2}{\omega^2}\left(1 + i\frac{\sigma_1}{\sigma_2}\right) \qquad \omega_{ps}^2 = \frac{n_s e^2}{\varepsilon_0 m} \qquad n_s = n_s(T, B, J)$$

Economou (1969)

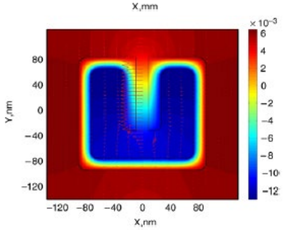
For $T < T_c$ and $\hbar\omega < 2\Delta$, $\frac{\sigma_1}{\sigma_2} << 1$

$R_{plasma} = U_{kinetic}/U_{magnetic}$



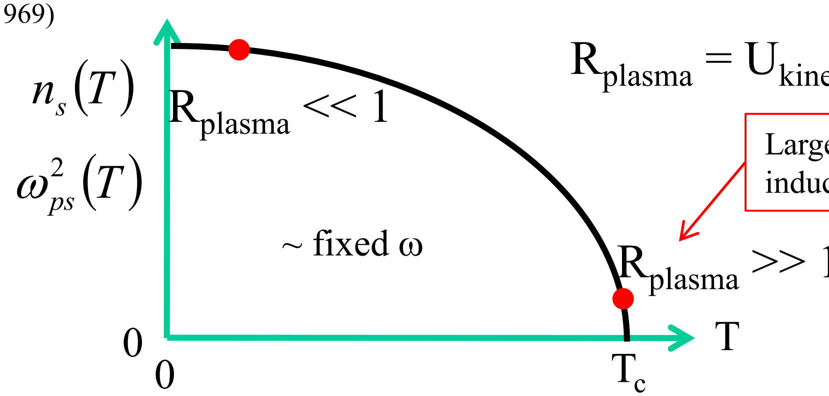
$T << T_c$

$R_{plasma} << 1$



T?T_c

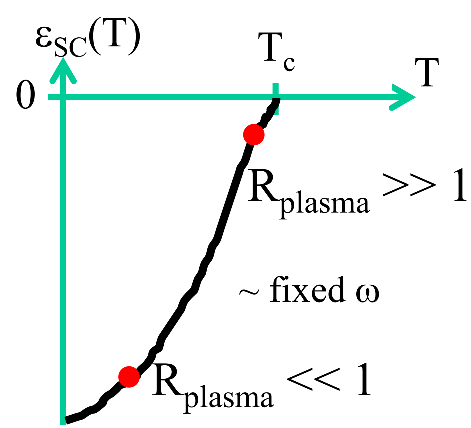
$R_{plasma} >> 1$



Large "kinetic inductance" L_{kin}

Penetration depth:

$$\lambda \sim \frac{\lambda_{EM}}{\sqrt{-\varepsilon_{SC}}}$$



Tuning the SC metamaterial plasma frequency:

Ricci, *et al.*, APL 88, 264102 (2006)

Ricci, *et al.*, IEEE TAS 17, 918 (2007)

SC plasmonics and extraordinary transmission:

Tsiatmas, *et al.*, APL 97, 111106 (2010)

Now investigate the plasmonic behavior of superconducting metamaterials …

When is Artificial Magnetism Lost Due to Losses?

Estimate: When transit time across unit cell ~ decay time due to losses

Metals $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$ $\frac{a_x}{v_g} > 1/\gamma$

a_x = unit cell size

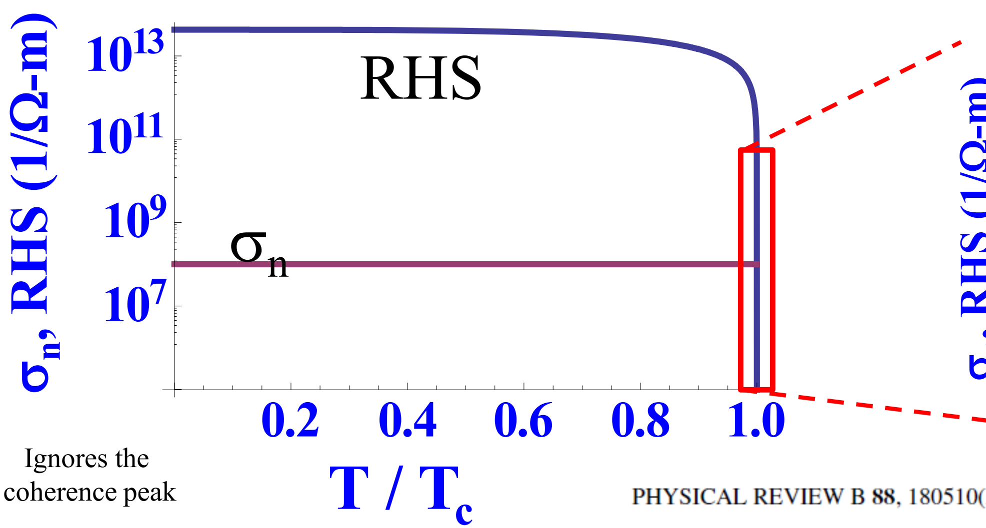
v_g = group velocity

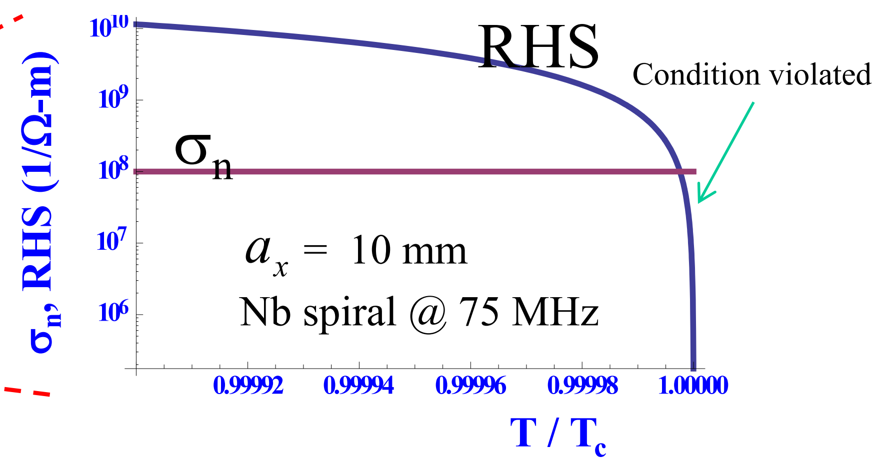
$$\frac{\gamma}{\omega} < \frac{1}{2\pi}\frac{\lambda_{EM}/a_x}{(1+R_{plasma})}$$

Superconductors $$\frac{\sigma_1}{\varepsilon_0\omega} < \frac{1}{2\pi}\frac{\lambda_{EM}/a_x}{(1+R_{plasma})}\left(\frac{\lambda_{EM}}{2\pi\,\lambda(T)}\right)^2$$

Condition to achieve artificial magnetism

λ(T) = magnetic penetration depth





Ignores the coherence peak

PHYSICAL REVIEW B **88**, 180510(R) (2013)

Plasmonic scaling of superconducting metamaterials

C. Kurter,[1] J. Abrahams,[1] G. Shvets,[2] and Steven M. Anlage[1]