A. Review of relevant concepts from Quantum Mechanics

Review of Basic Quantum (wave) Mechanics for *single* particles:
Time-dependent Schrodinger equation: $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{r})\psi$
Probability amplitude for finding the particle: $P(\vec{r},t) := \psi^*(\vec{r},t)\psi(\vec{r},t)$
Normalization condition on the wavefunction: $\int P(\vec{r},t)dV = \int \psi^*(\vec{r},t)\psi(\vec{r},t)dV = 1$ for all time t.
Probability current: $\vec{J}_{prob} = \frac{\hbar}{i2m}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) = Re[\psi^*\frac{\hbar}{im}\vec{\nabla}\psi]$. Note that $\vec{J}_{prob}$ has dimensions of inverse time.
Continuity equation for probability density: $\frac{\partial P}{\partial t} = -\vec{\nabla}\cdot\vec{J}_{prob}$
Charged particle under the influence of electric and magnetic fields, with associated scalar and vector potentials: $\vec{B} = \vec{\nabla}\times\vec{A}$ and $\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}$. The canonical momentum is the sum of the kinematic momentum and electromagnetic momentum: $m\vec{v} + q\vec{A}$.
Schrodinger equation including ϕ and $\vec{A}$: $i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}(\frac{\hbar}{i}\vec{\nabla} - q\vec{A})^2\psi + q\phi\psi$
Probability current including electromagnetic momentum $q\vec{A}$: $\vec{J}_{prob} = Re[\psi^*(\frac{\hbar}{im}\vec{\nabla} - \frac{q}{m}\vec{A})\psi]$.