Solution To PHYS622 Homework Set 8

1 Solution1

(a).

$$\Psi(\vec{x}) = (x+y+3z)f(r) = (r\sin\theta\cos\phi + r\sin\theta\sin\phi + 3r\cos\theta)f(r)$$
$$= \sqrt{\frac{4\pi}{3}} r f(r)[\frac{-1+i}{\sqrt{2}}Y_{1,+1} + \frac{1+i}{\sqrt{2}}Y_{1,-1} + 3Y_{1,0}]$$

Therefore $\Psi(\vec{x})$ is the eigenfuction of $\vec{L}^2$ with eigenvalue 1 .

(b).The three possible states are $m = 1, 0, -1$ Normalization condition $|\Psi| = \frac{44\pi}{3}|f(r)|^2$

$$P(m = 1, -1) = \frac{1}{11}, P(m = 0) = \frac{9}{11}$$

(c).Use Schrodinger equation in spherical coordinate,one has

$$V(r) = E + \frac{\hbar^2}{2m}\frac{4f'(r) + rf''(r)}{rf(r)}$$

2 Solution2

$$L_- Y_{1/2,1/2}(\theta,\psi) \propto -i\hbar e^{-i\phi}(-i\frac{\partial}{\partial\theta} - \cot\theta\frac{\partial}{\partial\phi})(e^{i\phi/2}\sqrt{\sin\theta}) = -\hbar e^{-i\phi/2}\frac{\cos\theta}{\sqrt{\sin\theta}}$$

However,using $L_- Y_{1/2,-1/2}(\theta,\psi) = 0$,one has $Y_{1/2,-1/2}(\theta,\psi) = e^{-i\phi/2}\sqrt{\sin\theta}$. Hence,two methods give contradictory results.

3 Solution3

$|j = 2, m = 2 >= |+, + >$
$|j = 2, m = 1 >= \frac{1}{\sqrt{2}}(|0, + > +|+, 0 >)$
$|j = 2, m = 0 >= \frac{1}{\sqrt{6}}(|-, + > +2|0, 0 > +|+, - >)$
$|j = 2, m = -1 >= \frac{1}{\sqrt{2}}(|0, - > +|-, 0 >)$
$|j = 2, m = -2 >= | - - >$
$|j = 1, m = 1 >= \frac{1}{\sqrt{2}}(|0, + > -|+, 0 >)$
$|j = 1, m = 0 >= \frac{1}{\sqrt{2}}(|-, + > -|+, - >)$
$|j = 1, m = -1 >= \frac{1}{\sqrt{2}}(|-, 0 > -|0, - >)$
$|j = 0, m = 0 >= \frac{1}{\sqrt{3}}(|+, - > -|0, 0 > +|-, + >)$

4 Solution4

(a).

$$\sum_{m=-j}^{m=j}|d^{j}_{mm'}(\beta)|^2 m=\sum_{m=-j}(<jm|e^{-iJ_y\beta/\hbar}|jm'>)(<jm|e^{-iJ_y\beta/\hbar}|jm'>)^* m$$
$$=<jm'|e^{iJ_y\beta/\hbar}J_z e^{-iJ_y\beta/\hbar}|jm'>=<jm'|J_z\cos\beta-J_x\sin\beta|jm'>=m'\cos\beta$$

For $j=1/2$,one has

$$d^j(\beta)=\begin{pmatrix}\cos\beta/2 & -\sin\beta/2\\ \sin\beta/2 & \cos\beta/2\end{pmatrix}$$
$$\sum_{m=-1/2}^{m=1/2}|d^{1/2}_{m,1/2}|^2 m=1/2\cos^2\beta/2-1/2\sin^2\beta/2=1/2\cos\beta$$
$$\sum_{m=-1/2}^{m=1/2}|d^{1/2}_{m,-1/2}|^2 m=-1/2\cos^2\beta/2-1/2\sin^2\beta/2=-1/2\cos\beta$$

(b).

$$\sum_{m=-j}^{m=j}|d^{j}_{mm'}(\beta)|^2 m^2=<jm'|e^{iJ_y\beta/\hbar}J_z^2 e^{-iJ_y\beta/\hbar}|jm'>$$
$$e^{iJ_y\beta/\hbar}J_z^2 e^{-iJ_y\beta/\hbar}=J_z^2\cos^2\beta-\sin\beta\cos\beta(J_xJ_z+J_zJ_x)+J_x^2\sin^2\beta$$
$$<jm'|e^{iJ_y\beta/\hbar}J_z^2 e^{-iJ_y\beta/\hbar}|jm'>=1/2j(j+1)\sin^2\beta+1/2m'^2(3\cos^2\beta-1)$$

5 Solution5

$$|J,m>=\sum_{m1,m2}|m1,m2><m1,m2|J,m>=\sum_{m2,m1}|m2,m1><m2,m1|J,m>$$

If the state is symmetric, $<m1,m2|J,m>=<m2,m1|J,m>$
If the state is anti-symmetric, $<m1,m2|J,m>=-<m2,m1|J,m>$
Using the hint,

$$<m1,m2|J,m>=(-1)^{j1+j2-J}<m2,m1|J,m>=(-1)^{2j-J}<m2,m1|J,m>=$$
$$(-1)^{2j+J}<m2,m1|J,m>=(-1)^J<m2,m1|J,m>$$

Therefore, J even,the state is symmetric; J odd,the state is anti-symmetric.