Solution To PHYS622 Homework Set 4

JJS 2.3,2.4,2.6,2.8,2.13,2.16

1 Solution1

(a).

$$\psi(0) = \begin{pmatrix} cos(\beta/2) \\ sin(\beta/2) \end{pmatrix}, \psi(t) = \begin{pmatrix} cos(\beta/2)e^{-i\omega t/2} \\ sin(\beta/2)e^{i\omega t/2} \end{pmatrix}$$

$$< S_x; +|\psi(t) >= \frac{1}{\sqrt{2}}(cos(\beta/2)e^{-i\omega t/2} + sin(\beta/2)e^{i\omega t/2})$$

$$| < S_x; +|\psi(t) > |^2 = \frac{1}{2}(1 + sin\beta cos\omega t)$$

(b).

$$< S_x >=< \psi(t)|S_x|\psi(t) >= \begin{pmatrix} cos(\beta/2)e^{-i\omega t/2} & sin(\beta/2)e^{i\omega t/2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} cos(\beta/2)e^{-i\omega t/2} \\ sin(\beta/2)e^{i\omega t/2} \end{pmatrix}$$

$$= \frac{\hbar}{2} sin\beta cos\omega t$$

(c).

$$\beta = 0, \psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, < S_x >= 0$$

$$\beta = \sigma/2, \psi(0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, < S_x >= \frac{\hbar}{2} cos\omega t$$

2 Solution2

$$\hat{x}(t) = \hat{x}(0) + \frac{\hat{p}(0)}{m} t$$

$$[\hat{x}(t), \hat{x}(0)] = [\frac{\hat{p}}{m} t, \hat{x}] = \frac{-i\hbar t}{m}$$

3 Solution3

$$H = \frac{\vec{p}^2}{2m} + V(\vec{x})$$

$$i\hbar \frac{d}{dt} < \vec{x} \cdot \vec{p} >= [\vec{x} \cdot \vec{p}, H] = [\vec{x}, H] \cdot \vec{p} + \vec{x} \cdot [\vec{x}, H] = i\hbar \frac{\vec{p}}{m} \cdot \vec{p} + (-i\hbar)\vec{x} \cdot \bigtriangledown V(\vec{x})$$

$$\frac{d}{dt} < \vec{x} \cdot \vec{p} >=< \frac{\vec{p}^2}{2m} > - < \vec{x} \cdot \bigtriangledown V >$$

For stationary states,i.e.eigenstates of H, $\frac{d}{dx} <\vec{x}\cdot\vec{p}>= 0,$ one gets $<T>=<\frac{\vec{p}^2}{2m}>=\frac{1}{2}<\vec{x}\cdot\bigtriangledown V>$.

4 Solution4

$$H=\begin{pmatrix}0 & \delta\\ \delta & 0\end{pmatrix}=\delta\sigma_x$$

(a).

$$|+>=\frac{1}{\sqrt{2}}(|a'>+|a''>),E=\delta$$
$$|->=\frac{1}{\sqrt{2}}(|a'>-|a''>),E=-\delta$$

(b).

$$|\psi(0)>=|a'>=\frac{1}{\sqrt{2}}(|+>+|->)$$
$$|\psi(t)>=\frac{1}{\sqrt{2}}(e^{-i\delta/\hbar}|+>+e^{i\delta/\hbar}|->)$$

(c).

$$|<a'|\psi(t)>|^2=sin^2\delta t/\hbar$$

(d).Spin $\frac{1}{2}$ system.

5 Solution5

(a).

$$\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}(a^++a),\hat{p}=i\sqrt{\frac{m\omega\hbar}{2}}(a^+-a).$$
$$<m|\hat{x}|n>=\sqrt{\frac{\hbar}{2m\omega}}(\sqrt{n}\delta_{n,m-1}+\sqrt{n+1}\delta_{m,n+1})$$
$$<m|\hat{p}|n>=i\sqrt{\frac{m\hbar\omega}{2}}(-\sqrt{n}\delta_{n,m-1}+\sqrt{n+1}\delta_{m,n+1})$$
$$<m|\hat{x}^2|n>=\frac{\hbar}{2m\omega}(\sqrt{(n+)(n+2)}\delta_{m,n+2}+\sqrt{n(n-1)}\delta_{m,n-2}+(2n+1)\delta_{m,n})$$
$$<m|\hat{p}^2|n>=-\frac{\hbar}{2m\omega}(\sqrt{(n+)(n+2)}\delta_{m,n+2}+\sqrt{n(n-1)}\delta_{m,n-2}-(2n+1)\delta_{m,n})$$

(b).

$$< n|\hat{T}|n >=< n|\frac{\hat{p}^2}{2m}n >= \frac{\hbar\omega}{4}(2n+1)$$
$$< n|\hat{V}|n >= \frac{1}{2}\omega^2 < \hat{x}^2 >= \frac{\hbar\omega}{4}(2n+1)$$
$$< \hat{T} >=< \hat{V} >$$

6 Solution6

(a).

$$|\psi >= cos\frac{\beta}{2}|0 > +sin\frac{\beta}{2}e^{i\alpha}|1 >$$
$$< x >= \sqrt{\frac{\hbar}{2m\omega}}sin\beta cos\alpha$$
$$\alpha = 0 \quad \beta = \frac{\pi}{2}, < x >\rightarrow max.$$
$$|\psi >= \frac{1}{\sqrt{2}}(|0 > +|1 >).$$

(b).

$$|\psi(t) >= \frac{1}{\sqrt{2}}(e^{-i\omega t/2}|0 > +e^{-i\omega 3t/2}|1 >)$$
$$(i). < x > |_s = \sqrt{\frac{\hbar}{2m\omega}}\psi(t)|(a^+ + a)|\psi(t) >= \sqrt{\frac{\hbar}{2m\omega}}cos\omega t.$$
$$(ii). < x > |_H =< \hat{x}(0) > cos\omega t + < \frac{\hat{p}(0)}{m\omega} > sin\omega t = \sqrt{\frac{\hbar}{2m\omega}}cos\omega t.$$

(c).

$$< \hat{x}^2(t) >= \frac{\hbar}{m\omega}$$
$$\Delta x = \sqrt{\hat{x}^2 - < x(t) >^2} = \sqrt{\frac{\hbar}{m\omega} - \frac{\hbar}{2m\omega}cos^2(\omega t)} = \sqrt{\frac{\hbar}{m\omega}}\sqrt{1 + sin^2\omega t}$$