Department of Physics
University of Maryland, College Park

Midterm Exam, Physics 622 — Friday, Oct. 7, 2005

Problem 1: Dirac Notation, Etc. (30 pts)

$|\phi_n\rangle$ are the eigenstates of a hermitian operator H. Assume that the states $|\phi_n\rangle$ form a discrete orthonormal basis. The operator $U(m,n)$ is defined by $U(m,n) = |\phi_m\rangle\langle\phi_n|$.

a) Calculate the hermitian conjugate $U^\dagger(m,n)$ of $U(m,n)$. (6 pts)

b) Calculate the commutator $[H, U(m,n)]$. (6 pts)

c) Calculate $\mathrm{Tr}U(m,n)$, the trace of the operator $U(m,n)$. (6 pts)

d) Calculate the trace $\mathrm{Tr}[AU^\dagger(p,q)]$. (6 pts)

e) Expand the operator A in terms of $U(m,n)$ and work out the coefficient of the expansion using bra-ket notation. (6 pts)

Problem 2: Time Evolution (35 pts)

Consider a system with two linearly independent states $|+\rangle$ and $-\rangle$. The most general state is a normalized linear combination $|\psi\rangle = a|+\rangle + b|-\rangle$ with $|a|^2 + |b|^2 = 1$. Suppose the hamiltonian matrix is,

$$H = \hbar \begin{pmatrix} \alpha & -i\beta \\ i\beta & \alpha \end{pmatrix} \tag{1}$$

where α and β are real constants.

a) Find the eigenvalues and normalized eigenvectors of the hamiltonian. (7 pts)

b) Suppose the system starts out at $|+\rangle$ at $t = 0$. What is the state at time t? Simplify your expression to get the full credit. (7 pts)

c) At what time is the system entirely in state $|-\rangle$? (7 pts)

d) If one measures the energy at t, what values will one find? With what probabilities? (7 pts)

e) Calculate the average energy at any time t. Calculate the standard deviation ΔE. (7 pts)

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Problem 3: One-Dimensional Harmonic Oscillator (35 pts)

Consider a particle subject to a one-dimensional simple harmonic oscillator potential and the hamiltonian is $H = \hat{p}^2/(2m) + (1/2)m\omega^2\hat{x}^2$.

a) Work out the Heisenberg equations of motion for $\hat{x}(t)$ and $\hat{p}(t)$. (7 pts)

b) Solve the above equations to find $\hat{x}(t)$ and $\hat{p}(t)$ in terms of $\hat{x}(0)$ and $\hat{p}(0)$ at $t = 0$. (7 pts)

c) If the oscillator is in state

$$|\psi\rangle = \exp(-i\hat{p}a/\hbar)|0\rangle \tag{2}$$

where $|0\rangle$ is the oscillator ground state and $\hat{p}$ is the momentum operator and a is some number with dimension of length, write down the coordinate space wave function of $|\psi\rangle$, in terms of $\langle x|0\rangle = \frac{1}{\sqrt{\sqrt{\pi}b}} \exp\left(-x^2/(2b^2)\right)$. (7 pts)

d) Calculate the commutator $[\hat{x}, e^{-i\hat{p}a/\hbar}]$. (7 pts)

e) Suppose at $t = 0$, the state vector is given by

$$\exp(-i\hat{p}a/\hbar)|0\rangle \tag{3}$$

Calculate the expectation value of $\langle\hat{x}\rangle$ for $t > 0$ using Heisenberg picture. Hint: use result from d) (7 pts)