Phys. 622: Problem Set V

1. This problem shows an alternative derivation of the WKB formula. Motivated by the free particle wave function, $\psi = A\exp(\pm ipx/\hbar)$, we write

$$\psi(x) = \exp(if(x)/\hbar) \tag{1}$$

where $f(x)$ is some complex function.

a. Put this in a stationary Schrodinger equation, and show that

$$i\hbar f'' - (f')^2 + p^2 = 0 \tag{2}$$

where $p = \sqrt{2m(E - V(x)}$.

b. Write $f(x)$ as a power series in $\hbar$

$$f(x) = f_0(x) + \hbar f_1(x) + \hbar^2 f_2(x) + ... \tag{3}$$

and, collecting like powers of $\hbar$, show that

$$(f_0')^2 = p^2, \quad if_0'' = 2f_0'f_1', \quad if_1'' = 2f_0'f_2' + (f_1')^2, ... \tag{4}$$

c. Solve for $f_0(x)$ and $f_1(x)$, and show that, to the first order in $\hbar$, you recover the WKB wave function.

2. Use the WKB approximation to find the allowed energies of the harmonic oscillator.

3. Use the WKB approximation to calculate the transmission probablity for a particle of energy E that encounters a finite square barrier of height $V_0 > E$ and width $2a$. Can you obtain the result in certain limit to the exact answer,

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right) \tag{5}$$

4. JJS: 2.23
5. JJS. 2.30
6. JJS. 2.31