Thus,

$$\langle L_z \rangle = \sum_{\substack{lm \\ l'm'}} F^{*}_{l'm'} F_{lm} \int Y^{*}_{l'm'} L_z Y_{lm}$$

$$= \sum_{\substack{ll' \\ mm'}} |F_{lm}|^2 m \, \delta_{ll'} \delta_{mm'}$$

$$= \sum_{lm} |F_{lm}|^2 m$$

$= 0$ because positive & negative m cancel.

12.
$$H = A \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}^2 + B \left[\frac{1}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^2 - \frac{1}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}^2 \right] \qquad (\hbar = 1)$$

$$= A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + B \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

$$= \begin{pmatrix} A & 0 & B \\ 0 & 0 & 0 \\ B & 0 & A \end{pmatrix}.$$

$$\text{Det} \begin{pmatrix} A-\lambda & 0 & B \\ 0 & -\lambda & 0 \\ B & 0 & A-\lambda \end{pmatrix} = \lambda \left((A-\lambda)^2 - B^2 \right) = 0$$

$$\lambda = 0, \qquad \lambda = A \pm B$$