

c. $\Theta |jm\rangle = i^{2m} |j, -m\rangle.$

Since. $\Theta = \eta e^{-i\pi J_y/\hbar} K$

$$\begin{aligned}\Theta |jm\rangle &= \eta e^{-i\pi J_y/\hbar} |jm\rangle \\ &= \eta \sum_{m'} D_{m'm}(\pi) |jm'\rangle.\end{aligned}$$

$$\begin{aligned}D_{m'm}(\pi) &= (-1)^{j+m} \delta_{m', -m} \\ &\downarrow \\ &= \eta \sum_{m'} \delta_{m', -m} (-1)^{j+m} |jm'\rangle\end{aligned}$$

~~Thus,~~ $= \eta (-1)^{j+m} |j, -m\rangle$

thus, we choose $\eta = (-1)^j$

$$\Theta |jm\rangle = (-1)^m |j, -m\rangle$$

11. $\psi = \sum_{lm} F_{lm} Y_{lm}(\theta, \phi)$

For non-degenerate state, ψ is real.

$$\begin{aligned}F_{lm} Y_{lm}(\theta, \phi) + F_{l-m} Y_{l-m}(\theta, \phi) \\ = F_{lm}^* Y_{lm}^*(\theta, \phi) + F_{l-m}^* Y_{l-m}^*(\theta, \phi).\end{aligned}$$

$$= (-1)^m F_{lm}^* Y_{l-m}(\theta, \phi) + F_{l-m}^* (-1)^m Y_{lm}(\theta, \phi).$$

thus. $F_{lm} = F_{l-m}^* (-1)^m$