

9. it is $\phi^{\dagger}(-\vec{p}')$.

$$\phi(\vec{p}') = \langle \vec{p}' | \alpha \rangle$$

$$\begin{aligned} \phi(\vec{p}') &= \langle \vec{p}' | \theta | \alpha \rangle^{\dagger} \\ &= \langle -\vec{p}' | \theta \alpha \rangle^{\dagger} \end{aligned}$$

$$\text{or } \phi^{\dagger}(-\vec{p}) = \langle \vec{p}' | \theta \alpha \rangle$$

10. a) $\hat{D}(\mathbf{R}) |j m\rangle$

$$= \sum_{m'} |j m'\rangle D_{m'm}(\mathbf{R}).$$

$$\begin{aligned} \theta \hat{D}(\mathbf{R}) |j m\rangle &= \sum_{m'} D_{m'm}^{\dagger}(\mathbf{R}) \theta |j m'\rangle \\ &= \sum_{m'} D_{m'm}^{\dagger}(\mathbf{R}) (i)^{2m'} |j - m'\rangle \end{aligned}$$

On the other hand

$$\theta \hat{D}(\mathbf{R}) |j m\rangle$$

$$= \hat{D}(\mathbf{R}) \theta |j m\rangle$$

$$= \sum_{m'} D_{m'm}(\mathbf{R}) (-1)^{m'} |j - m'\rangle$$

$$= \hat{D}(\mathbf{R}) (-1)^m |j - m\rangle$$

$$= \sum_{m'} D_{m'-m}(\mathbf{R}) (-1)^m |j - m'\rangle$$

(because $e^{i\vec{\theta} \cdot \vec{J}}$ does not change $i \rightarrow -i$, $\vec{J} \rightarrow -\vec{J}$).

Compare. $D_{m'm}^{\dagger}(\mathbf{R}) = (-1)^{m-m'} D_{-m'-m}(\mathbf{R})$