7: a) $\psi(\vec{x},t) = e^{-iEt + i\vec{p}\cdot\vec{x}}$ $\left(E = p^2/2m\right)$

$\psi^*(\vec{x}, -t) = e^{-iEt - i\vec{p}\cdot\vec{x}}$

which is a plane wave with momentum $\vec{p}$

b)

$$\chi_+(\vec{n}) = \begin{pmatrix} \cos\frac{\beta}{2} \\ e^{i\alpha}\sin\frac{\beta}{2} \end{pmatrix}.$$

$$-i\sigma_2\chi_+^*(\vec{n}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\frac{\beta}{2} \\ e^{-i\alpha}\sin\frac{\beta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -e^{-i\alpha}\sin\frac{\beta}{2} \\ \cos\frac{\beta}{2} \end{pmatrix}.$$

On the other hand, when spin direction is reversed

$$\beta \to \pi - \beta$$
$$\alpha \to \pi + \alpha.$$

$$\cos\left(\frac{\pi}{2} - \frac{\beta}{2}\right) = +\sin\frac{\beta}{2}$$
$$\sin\left(\frac{\pi}{2} - \frac{\beta}{2}\right) = \cos\frac{\beta}{2}$$

$$\chi_-(\vec{n}) = \begin{pmatrix} +\sin\frac{\beta}{2} \\ -e^{i\alpha}\cos\frac{\beta}{2} \end{pmatrix}$$

$$= \left(-e^{i\alpha}\right)\begin{pmatrix} -e^{-i\alpha}\sin\beta/2 \\ \cos\beta/2 \end{pmatrix} = -e^{i\alpha}\left(-i\sigma_2\right)\chi_+^*(\vec{n})$$