Physics 603, Spring 2003, O.W. Greenberg

Solutions to homework assignment 1, due Feb. 11, 2003

1. Make a classical calculation for free, nonrelativistic, structureless particles to show that $S = \hat{S}(N, V^{2/3}E)$.
Use $S = k ln\Omega(N, V, E)$, where Ω is the number of microstates subject to the constraints $p \leq p_{max}$

$$N = h^{-3}\int d^3p d^3x = \frac{4\pi}{3h^3}p_{max}^3 V, \qquad (1)$$

$$E = h^{-3}\int d^3p d^3x \frac{p^2}{2m} = \frac{2\pi}{10mh^3}p_{max}^5 V. \qquad (2)$$

Eliminate p_{max}. Then Ω has the form

$$\Omega(N, V, E) = \Omega(N, V, E = const. N^{5/3}V^{-2/3}), \qquad (3)$$

so Ω depends only two variables and we can define a *new* function $\hat{\Omega}(N, V^{2/3}E)$ (or a different new function $\tilde{\Omega}(V, N^{-5/3}E)$). Then S has the form given in the problem.

2. Use a quantum argument to show that $S = \hat{S}(N, V^{1/3}E)$ for free, massless, relativistic particles. From this result, derive the relation between P, E and V for a radiation gas.

Need the number of quantum states subject to the total number being N and the total energy being E. Then

$$E = 2\sum_i c|\mathbf{p}_i| \qquad (4)$$

where the factor 2 is for the two degrees of polarization of photons for the radiation gas, and $\mathbf{p}_i = \frac{h}{2L}\sqrt{n_{ix}^2, n_{iy}^2, n_{iz}^2}$. If the photons are in a box of side L, $p_x = (h/2L)n_x$, etc., then $E = \sum_i c\mathbf{p}_i$ scales as $L^{-1} = V^{-1/3}$, so $EV^{1/3} = const.$ Then the argument of 1. shows that $\Omega(N, V, E) = \hat{\Omega}(N, V^{1/3}E)$. From $P = -\frac{\partial E}{\partial V}|_{N,S}$ we get

$$P = \frac{E}{3V}. \qquad (5)$$