

Write clearly and show all work leading to your results. Write on only one side of the page.

If you want to know your grade before receiving the registrar's report, fill out a card with your name and the labels: homework score, final exam score and semester grade. I will put the card with your grades in your Physics Department mail box.

1. Consider a system with N of non-interacting particles, each at a different site. Each particle can be in one of two energy levels, $\pm\epsilon$, $\epsilon > 0$. Assume N is large.

(a) Calculate the number of microstates Ω in terms of N , N_- and N_+ at fixed energy E .

(b) Calculate the entropy $S(N, E)$.

(c) Plot S/Nk vs $E/N\epsilon$.

(d) Plot $1/kT$ vs $E/N\epsilon$.

(e) For what range of E is $T > 0$?

(f) Interpret the range in which $(\partial S/\partial E)_N \leq 0$.

(g) Calculate the pressure $P(N, E)$.

(h) Calculate C_V as a function of $x = kT/\epsilon$ and sketch it for $T > 0$.

(i) Extra credit: Calculate the small- x and large- x behavior.

2. Consider a non-interacting non-relativistic Fermi gas of N particles of mass m and spin S in volume L^D in D space dimensions.

Choose from $\langle n_\epsilon \rangle = 1/(\exp(\beta(\epsilon - \mu)) \pm 1)$ for your calculations. Use

Ω_D = volume of the unit sphere in D dimensions.

(a) Plot $\langle n_\epsilon \rangle$ vs ϵ/kT for $T \rightarrow 0^+$.

(b) Write N as an integral over the magnitude of \mathbf{p} in D dimensions; then change variables to a dimensionless variable so that your integral depends only on z . Hint: use dimensional analysis to decide how h enters.

(c) Estimate the increase in energy for T just above 0^+ by estimating the fraction of particles excited and the average excitation energy of the excited particles.

- (d) Find the T dependence of C_V/Nk near 0^+ .
- (e) Find C_V/Nk for $T \rightarrow \infty$. Hint: use a general theorem, not a detailed calculation.

3. The one-dimensional Ising model has the Hamiltonian

$$H\{\sigma_i\} = -J \sum_{n.n.} \sigma_i \sigma_j - \mu B \sum_i \sigma_i,$$

where σ_i is the spin at the i th site, J is the coupling parameter, μ is the magnetic moment of the spins, B is the external magnetic field and $n.n.$ stands for the sum over nearest neighbors.

- (a) Rewrite the Hamiltonian for N sites as a torus with the ends identified.
- (b) Write the formula for the partition function.
- (c) Define the matrix elements of an operator \mathbf{P} between neighboring sites, $\langle \sigma_i | \mathbf{P} | \sigma_{i+1} \rangle$, so that the partition function is the trace of \mathbf{P}^N .
- (d) Calculate the eigenvalues of \mathbf{P} .
- (e) Assuming N is large, calculate the partition function.
- (f) Calculate the internal energy for $B = 0$.
- (g) Graph the internal energy U_0/NJ as a function of kT/J . (h) Extra credit: give the limiting behaviors for $T \rightarrow 0$ and $T \rightarrow \infty$.
- (i) Extra credit: Give the physical interpretation of the limiting values for $T \rightarrow 0$ and $T \rightarrow \infty$.
- (j) Extra credit: Calculate the magnetization as a function of β .
- (k) Extra credit: Is there a ferromagnetic phase transition and, if so, what is T_c ?