

Write clearly and show all work leading to your results. Write on only one side of the page.

1. (a) Graph  $C_V/Nk$  for  $HD$  in the range 10 – 5000 K. Show the values of  $C_V/Nk$  corresponding to the excitation of various combinations of the vibrational, translational and rotational modes. Show where room temperature is.

(b) Write the wavefunction of a homonuclear diatomic molecule  $(Z^A)_2$  as a product of factors corresponding to the center-of-mass, electronic, rotational, vibrational and nuclear spin wavefunctions. Use  $\mathbf{r}$  for space coordinates and  $\sigma$  for spin coordinates. Show which factors depend on which coordinates.

(c) Explain why the symmetry under exchange of the nuclei is the same as the symmetry under the product of the symmetry of the rotational states under exchange of the nuclei and the symmetry under exchange of the nuclear spins for almost all diatomic molecules.

(d) The  $N^{14}$  nucleus has spin  $\hbar$  and charge 7 in units of the proton charge. Calculate the factors in the partition function connected with the nuclei under the assumption that the nitrogen nucleus is made of 7 electrons and 14 protons and compare with the result if the nucleus is made of 7 neutrons and 7 protons. In each case, state which type of molecule will be present at low  $T$ .

2. (a) Given that the grand partition function for an ideal gas of spin 0 nonrelativistic bosons is

$$\mathcal{Q} = \prod \frac{1}{1 - ze^{-\beta\epsilon}}, \quad (1)$$

use the standard phase space formula  $h^{-3}d^3pd^3q$  to find the total number of particles in volume  $V$  as an integral over energy, paying special attention to the zero energy state.

(b) Use your result in (a) to find the condition for Bose-Einstein condensation in an ideal bose gas; i.e., find the temperature  $T_c$  at which the condensate starts to form.

(c) Find the form of the temperature dependence of  $C_V$  for an ideal bose gas for  $T < T_c$ . Don't go through the whole calculation here; just figure out how  $T$  enters in the equation for the energy and then get the  $T$  dependence of  $C_V$ .