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Use Theorem 41 to write

$$\left.\begin{aligned} u(\hat{n}_1, \theta_1) &= \sigma_0 \cos\frac{\theta_1}{2} - i\,\hat{n}_1\cdot\vec{\sigma}\sin\frac{\theta_1}{2} \\ u(\hat{n}_2, \theta_2) &= \sigma_0 \cos\frac{\theta_2}{2} - i\,\hat{n}_2\cdot\vec{\sigma}\sin\frac{\theta_2}{2} \end{aligned}\right\} \Rightarrow$$

$$u(\hat{n}, \theta) = u_1\, u_2 =$$

$$\left\{\sigma_0\cos\frac{\theta_1}{2} - i\,\hat{n}_1\cdot\vec{\sigma}\sin\frac{\theta_1}{2}\right\}\left\{\sigma_0\cos\frac{\theta_2}{2} - i\,\hat{n}_2\cdot\vec{\sigma}\sin\frac{\theta_2}{2}\right\} =$$

$$\sigma_0\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - i\,\hat{n}_2\cdot\vec{\sigma}\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2} - i\,\hat{n}_1\cdot\sigma\cos\frac{\theta_2}{2}\sin\frac{\theta_1}{2}$$

$$- \hat{n}_1\cdot\hat{n}_2\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} - i(\hat{n}_1\times\hat{n}_2)\cdot\vec{\sigma}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}.$$

[Here we used $(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = \vec{a}\cdot\vec{b}\,\sigma_0 + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$]

But, $u(\hat{n}, \theta) = \cos\frac{\theta}{2}\sigma_0 - i\,\hat{n}\cdot\vec{\sigma}\sin\frac{\theta}{2}$.

Comparing terms $\Rightarrow$

i) $$\boxed{\cos\frac{\theta}{2} = \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - \hat{n}_1\cdot\hat{n}_2\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}}$$

ii) $$\boxed{\hat{n}\sin\frac{\theta}{2} = \hat{n}_1\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2} + \hat{n}_2\sin\frac{\theta_2}{2}\cos\frac{\theta_1}{2} + (\hat{n}_1\times\hat{n}_2)\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}}$$

Let $\vec{\tau} = \hat{n}\tan\frac{\theta}{2}$, etc. Then ii) $\div$ i) $\Rightarrow$

$$\boxed{\vec{\tau} = \frac{\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_1\times\vec{\tau}_2}{1 - \vec{\tau}_1\cdot\vec{\tau}_2}}$$

Combining rotations is complicated!