a) Note that we can write $M(R,\alpha)$ as the product $M(R\alpha) = M(I,\alpha)\, M(R,0)$.

See "relation" 1 of Theorem 35. It is easy to write each factor in exponential form which, by theorem 37, is enough.

We have $M(R,0) = \left(\begin{array}{ccc|c} & R & & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$. Let $R = R(\hat{n},\theta)$, & let $\mathcal{J}_j \stackrel{\text{def}}{=} \left(\begin{array}{ccc|c} & J_j & & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 0 \end{array}\right)$

Then, $\boxed{e^{\theta \hat{n}\cdot\vec{\mathcal{J}}} = \left(\begin{array}{ccc|c} & R(\hat{n},\theta) & & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right) = M(R,0)}$

Next work on $M(I,\alpha) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \vec{\alpha} \\ 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$

Consider the matrix

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Then, $P_1^2 = 0$, & $e^{\alpha_1 P_1} = I + \alpha_1 P_1$!

Thus, $\boxed{M(I,\alpha) = e^{\vec{\alpha}\cdot\vec{P}}}$ where

P_1 = already defined

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

b) $\vec{\mathcal{J}}$'s + $\vec{P}$'s form a basis with C.R.:

$$\left.\begin{aligned} [\mathcal{J}_j, \mathcal{J}_k] &= \sum_l \epsilon_{jkl} \mathcal{J}_l \\ [\mathcal{J}_j, P_k] &= \sum_l \epsilon_{jkl} P_l \\ [P_j, P_k] &= 0 \end{aligned}\right\}$$

Relations obtained using the matrices above.

Lie Algebra has dimension 6.