b) $\vec{J}^2 = \sum \vec{J}_j^2$. $J_1^2 = \begin{pmatrix} 0&0&0\\0&0&-1\\0&1&0 \end{pmatrix}^2 = \begin{pmatrix} 0&0&0\\0&-1&0\\0&0&-1 \end{pmatrix}$

$J_2^2 = \begin{pmatrix} 0&0&1\\0&0&0\\-1&0&0 \end{pmatrix}^2 = \begin{pmatrix} -1&0&0\\0&0&0\\0&0&-1 \end{pmatrix}$, $J_3^2 = \begin{pmatrix} 0&-1&0\\1&0&0\\0&0&0 \end{pmatrix}^2 = \begin{pmatrix} -1&0&0\\0&-1&0\\0&0&0 \end{pmatrix}$

$\therefore \boxed{\vec{J}^2 = \begin{pmatrix} -2 & & 0\\ & -2 & \\ 0 & & -2 \end{pmatrix} = -j(j+1)I \text{ with } j=1 \text{ as expected}}$

For $\vec{\mathcal{J}}^2$ the result is

$$\boxed{\vec{\mathcal{J}}^2 = \sum \mathcal{J}_i^2 = \sum \left(-\tfrac{i}{2}\right)^2 \sigma_j^2 = -\tfrac{3}{4}\begin{pmatrix} 1&0\\0&1 \end{pmatrix} = -j(j+1)\sigma_0 \text{ with } j=\tfrac{1}{2}}$$

c) Using part a, since $R(\hat{e}_3,\theta) = \sum \frac{(\theta J_3)^n}{n!}$, the eigenvalues of $R$ are $e^{\lambda\theta}$ where $\lambda$ are eigenvalues of $J_3$, and eigenvectors are the same.

| | | |
|---|---|---|
| $\lambda=0 \Rightarrow e^{\lambda\theta}=1$, $\hat{e}_3$ | $\lambda=i \Rightarrow e^{\lambda\theta}=e^{i\theta}$, $\hat{e}_1 - i\hat{e}_2$ | $\lambda=-i \Rightarrow e^{-i\theta}$ $\hat{e}_1 + i\hat{e}_2$ |

Similarly for $u(\hat{e}_3,\theta)$

| | |
|---|---|
| $e^{-i\theta/2}$, $\begin{pmatrix}1\\0\end{pmatrix}$ | $e^{i\theta/2}$, $\begin{pmatrix}0\\1\end{pmatrix}$ |