Dragt 79

a) $J_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\det(J_3 - \lambda I) = \begin{vmatrix} -\lambda & -1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix}$

$= -\lambda(\lambda^2+1) \Rightarrow \boxed{\text{eigenvalues} = 0, +i, -i}$

Eigen vector with eigenvalue 0 is $\hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

To get others, solve

$$\begin{aligned} -b &= \pm ia \\ a &= \pm ib \end{aligned} \Rightarrow \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

$\therefore$ $\boxed{\hat{e}_3 \text{ has } \lambda = 0, \quad (\hat{e}_1 - i\hat{e}_2) \text{ has } \lambda = i, \quad \hat{e}_1 + i\hat{e}_2 \text{ has } \lambda = -i}$

Note that eigenvalues of iJ_3 are $-1, 0, 1$.
Therefore, j has the value $j = 1$.

Next work with $\mathcal{J}_3$:

By inspection

$$\mathcal{J}_3 = -\tfrac{1}{2} i \sigma_3 = \begin{pmatrix} -i/2 & 0 \\ 0 & i/2 \end{pmatrix}$$

$\therefore$ $\boxed{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ has } \lambda = -i/2 \quad + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ has } \lambda = \tfrac{i}{2}}$

Therefore $i\mathcal{J}_3$ has eigenvalues $-1/2$ and $1/2$.

Consequently $j = 1/2$.