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a) $\text{tr}\,\sigma_j = 0$ for $j = 1$ to 3 by inspection

b) $\text{tr}\,\sigma_0 \sigma_j = \text{tr}\,\sigma_j = 0$ for $j = 1$ to 3

$\text{tr}\,\sigma_0 \sigma_0 = \text{tr}\,\sigma_0 = 2$

$\therefore \text{tr}\,\sigma_0 \sigma_j = 2\delta_{0j}$ for $j = 0$ to 3

Also, $\sigma_1 \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \Rightarrow$

$\boxed{\sigma_1 \sigma_2 = i\sigma_3}$ cyclicly, and

$\sigma_1 \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sigma_0$, etc.

$\therefore \text{tr}\,\sigma_j \sigma_j = 2 \Rightarrow \text{tr}\,(\sigma_j \sigma_k) = 2\delta_{jk}$

for $j, k = 1$ to 4

c) Observe that $\sigma_j^{\dagger} = \sigma_j$. It follows that

$\sigma_2 \sigma_1 = (\sigma_1 \sigma_2)^{\dagger} = (i\sigma_3)^{\dagger} = -i\sigma_3 = -\sigma_1\sigma_2$