e) $\sum_k \epsilon_{ijk} \epsilon_{lmk} = 0$ unless there is "overlap" of i, j with l, m since otherwise at least some $\epsilon_{\dots}$ factor is always zero.

If $i = l$ and $j = m$, then we get $\epsilon^2_{\dots}$, which for one term in the sum is $= 1$.

If $i = m$ and $j = l$, then we get $-\epsilon^2_{\dots}$ which for one term in the sum is $= -1$.

Thus we have

$$\boxed{\sum_k \epsilon_{ijk} \epsilon_{lmk} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}}$$

Suppose we write

$$\vec{b} \times \vec{c} = \sum_{ijk} \epsilon_{ijk} b_i c_j \hat{e}_k$$

and $\vec{a} = \sum_l a_l \hat{e}_l$. Then

$$\vec{a} \times (\vec{b} \times \vec{c}) = -(\vec{b} \times \vec{c}) \times \vec{a}$$

$$= -\sum_{ijkl} \epsilon_{ijk} b_i c_j a_l \hat{e}_k \times \hat{e}_l$$