

Differentiating both sides \Rightarrow

$$-e^{-\theta J_2} J_2 J_1 e^{\theta J_2} + e^{-\theta J_2} J_1 J_2 e^{\theta J_2} = \sum_1^{\infty} \frac{\theta^{n-1}}{(n-1)!} \Omega_n$$

$$\text{or } -J_2 \left(e^{-\theta J_2} J_1 e^{\theta J_2} \right) + \left(e^{-\theta J_2} J_1 e^{\theta J_2} \right) J_2 = \sum_0^{\infty} \frac{\theta^n}{n!} \Omega_{n+1}$$

Evaluating the left hand side using the original expansion \Rightarrow

$$\sum_0^{\infty} \frac{\theta^n}{n!} [\Omega_n J_2 - J_2 \Omega_n] = \sum_0^{\infty} \frac{\theta^n}{n!} \Omega_{n+1}$$

$$\text{or } \Omega_{n+1} = [\Omega_n J_2 - J_2 \Omega_n] = -[J_2, \Omega_n].$$

This along with the fact that $\Omega_0 = J_1$ determines all the Ω_n . We get

$$\Omega_0 = J_1$$

$$\Omega_1 = [\Omega_0 J_2 - J_2 \Omega_0] = [J_1 J_2 - J_2 J_1] = J_3$$

$$\Omega_2 = [\Omega_1 J_2 - J_2 \Omega_1] = [J_3 J_2 - J_2 J_3] = -J_1$$

$$\Omega_3 = [\Omega_2 J_2 - J_2 \Omega_2] = [-J_1 J_2 + J_2 J_1] = -J_3$$

$$\Omega_4 = [\Omega_3 J_2 - J_2 \Omega_3] = [-J_3 J_2 + J_2 J_3] = J_1$$

etc.