Now $J_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \Rightarrow J_1 J_2 - J_2 J_1 = J_3$

So we have verified that

$$e^{-\theta J_2} J_1 e^{\theta J_2} = J_1 \cos\theta + \underbrace{[J_1 J_2 - J_2 J_1]}_{= J_3} \sin\theta$$

The other cases are similar.

Note that the formula above is very similar to the formula

$$e^{-\theta J_2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ 0 \\ \sin\theta \end{pmatrix} = \cos\theta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

or $$e^{-\theta J_2} \hat{e}_1 = \cos\theta \, \hat{e}_1 + \sin\theta \, \hat{e}_3$$

Method II. Assume that

$$e^{-\theta J_2} J_1 e^{\theta J_2} = \sum_0^\infty \frac{\theta^n}{n!} \Omega_n \text{ where the}$$

Ω_n are <u>matrices yet to be determined</u>.

Putting $\theta = 0 \Rightarrow J_1 = \Omega_0$