(b) $\sigma_1\sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$\sigma_2\sigma_1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

$\sigma_1\sigma_2 - \sigma_2\sigma_1 = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\,\sigma_3$

One finds in general

$[\sigma_j, \sigma_k] = (2i)\sum_\ell \epsilon_{jk\ell}\,\sigma_\ell$. Now let

$J_j = (-\frac{i}{2})\,\sigma_j$. Then,

$[J_j, J_k] = (-\frac{i}{2})^2[\sigma_j, \sigma_k] = (-\frac{i}{2})^2(2i)\sum_\ell \epsilon_{jk\ell}\,\sigma_\ell$

$= (-\frac{i}{2})(2i)\sum_\ell \epsilon_{jk\ell}\,(-\frac{i}{2})\,\sigma_\ell = \sum_\ell \epsilon_{jk\ell}\,J_\ell$.

So, the J's obey same commutation rules as the J's.

Also note some other important properties of the Pauli matrices:

a) $\sigma_i\sigma_j + \sigma_j\sigma_i = 0 \quad (i \neq j)$ -- anticommuting property.

b) $\sigma_j\sigma_k = i\sum_\ell \epsilon_{jk\ell}\,\sigma_\ell + \delta_{jk}\,\sigma_0$

where $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$