$$\begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_1\cos\theta + a_3\sin\theta \\ a_2 \\ -a_1\sin\theta + a_3\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} b_1' \\ b_2' \\ b_3' \end{pmatrix} = \begin{pmatrix} b_1\cos\theta + b_3\sin\theta \\ b_2 \\ -b_1\sin\theta + b_3\cos\theta \end{pmatrix}$$

$$c_3' = \sum_{jk} \varepsilon_{3jk} a_j' b_k' = a_1' b_2' - a_2' b_1' = a_1 b_2 \cos\theta + a_3 b_2 \sin\theta$$

$$-a_2 b_1 \cos\theta - a_2 b_3 \sin\theta = (a_1 b_2 - a_2 b_1)\cos\theta - (a_2 b_3 - a_3 b_2)\sin\theta$$

$= c_3 \cos\theta - c_1 \sin\theta$. Similarly,

$c_2' = c_2$ , $c_1' = c_1 \cos\theta + c_3 \sin\theta$. Also,

$$(R_{ij})(c_j) = \begin{pmatrix} c_1\cos\theta + c_3\sin\theta \\ c_2 \\ -c_1\sin\theta + c_3\cos\theta \end{pmatrix} = \begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix}, \text{ and } \det R = 1.$$

We have proved that the formula $c_i' = (\det R) \sum_j R_{ij} c_j$ is true for inversion in the origin, and for rotations about the $x_1$, $x_2$ and $x_3$ axes.

Since any arbitrary orthogonal transformation can be obtained by applying these operations in succession, the theorem is proved.