This gives
$$c_1' = \sum_{j,k} \varepsilon_{1jk}\, a_j'\, b_k' = a_2' b_3' - a_3' b_2'$$
$$= a_1 b_3 \sin\theta + a_2 b_3 \cos\theta - a_3 b_1 \sin\theta - a_3 b_2 \cos\theta$$
$$= (a_2 b_3 - a_3 b_2)\cos\theta + (a_1 b_3 - a_3 b_1)\sin\theta$$
$$= c_1 \cos\theta - c_2 \sin\theta$$

Similarly
$$c_2' = c_1 \sin\theta + c_2 \cos\theta$$
$$c_3' = c_3$$

Also, $\det R = \cos^2\theta + \sin^2\theta = 1$

In addition,
$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} c_1\cos\theta - c_2\sin\theta \\ c_1\sin\theta + c_2\cos\theta \\ c_3 \end{pmatrix} = \begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix}$$

$$\therefore\ c_i' = \det R \sum_j R_{ij}\, c_j$$

(c) rotation about x_1 axis

$$(R_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2\cos\theta - a_3\sin\theta \\ a_2\sin\theta + a_3\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} b_1' \\ b_2' \\ b_3' \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2\cos\theta - b_3\sin\theta \\ b_2\sin\theta + b_3\cos\theta \end{pmatrix}$$