Begin with $c_i = \sum_{j,k} \varepsilon_{ijk} b_j c_k$. Suppose that

$$a_i' = \sum_j R_{ij} a_j \qquad b_i' = \sum_j R_{ij} b_j \qquad c_i' = \sum \varepsilon_{ijk} a_j' b_k'$$

Show that $c_i' = (\det R) \sum_j R_{ij} c_j$

Proof: We can verify this result for:

(a) inversion in origin

$$R = -I \qquad \therefore a_i' = -a_i \qquad b_i' = -b_i$$

$$c_i' = \sum_{j,k} \varepsilon_{ijk} a_j' b_k' = \sum_{j,k} \varepsilon_{ijk} (-a_j)(-b_k) = \sum_{j,k} \varepsilon_{ijk} a_j b_k$$
$$= c_i$$

Now, look at $c_i' \overset{?}{=} (\det R) \sum_j R_{ij} c_j$

$$\det R = \det(-I) = -1 \qquad \sum_j R_{ij} c_j = -c_i$$

$$\therefore (\det R) \sum_j R_{ij} c_j = (-1)(-c_i) = c_i = c_i'$$

(b) Rotation about x_3 axis

$$(R_{ij}) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a_i' = \sum_j R_{ij} a_j \qquad b_i' = \sum_j R_{ij} b_j$$

$$a_1' = a_1 \cos\theta - a_2 \sin\theta \qquad b_1' = b_1 \cos\theta - b_2 \sin\theta$$

$$a_2' = a_1 \sin\theta + a_2 \cos\theta \qquad b_2' = b_1 \sin\theta + b_2 \cos\theta$$

$$a_3' = a_3 \qquad b_3' = b_3$$