Also, $\sin\theta_1 = \sin(120^\circ) = \frac{\sqrt{3}}{2}$. Therefore

$$2\hat{a}\cdot\vec{J}\sin\theta_1 = 2(a_1J_1 + a_2J_2 + a_3J_3)\sin\theta_1$$

$$= \frac{\sqrt{3}}{2}\cdot 2\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -a_1 \\ 0 & a_1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & a_2 \\ 0 & 0 & 0 \\ -a_2 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -a_3 & 0 \\ a_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$= \sqrt{3}\begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \quad (2).$$

Comparing (1) and (2) $\Rightarrow$

$-\sqrt{3}\,a_3 = -1$, $\sqrt{3}\,a_2 = 1$, $-\sqrt{3}\,a_1 = -1$ $\Rightarrow$ $\boxed{\text{Axis of rotation} \quad \hat{a} = \frac{1}{\sqrt{3}}(\hat{e}_1 + \hat{e}_2 + \hat{e}_3)}$

<u>Next work on $R_{\hat{b}}(\theta_2)$</u>: One finds $\mathrm{tr}\, R_{\hat{b}}(\theta_2) = 0$

$\Rightarrow 1 + 2\cos\theta_2 = 0 \Rightarrow \boxed{\theta_2 = 120^\circ}$. Also,

$$R_{\hat{b}}(\theta_2) - \tilde{R}_{\hat{b}}(\theta_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

In addition, $2\hat{b}\cdot\vec{J}\sin\theta_2 = 2\frac{\sqrt{3}}{2}\,\vec{b}\cdot\vec{J} = \sqrt{3}\begin{pmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{pmatrix}$.

Comparing entries gives

$-\sqrt{3}\,b_3 = 1$, $\sqrt{3}\,b_2 = 1$, $-\sqrt{3}\,b_1 = -1$ $\Rightarrow$

$$\boxed{\hat{b} = \frac{1}{\sqrt{3}}(\hat{e}_1 + \hat{e}_2 - \hat{e}_3) \qquad \text{Axis of rotation}}$$