(a) $R_{\hat{e}_1}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ by problem DMNR 2

$$R_{\hat{e}_1}\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{\hat{e}_2}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_{\hat{e}_2}\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$R_{\hat{e}_1}\left(\tfrac{\pi}{2}\right) R_{\hat{e}_2}\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = R_{\hat{a}}(\theta_1) \text{ say.}$$

$$R_{\hat{e}_2}\left(\tfrac{\pi}{2}\right) R_{\hat{e}_1}\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} = R_{\hat{b}}(\theta_2) \quad \text{say}$$

(b) $\mathrm{tr}\, R_{\hat{a}}(\theta) = (1 + 2\cos\theta)$. For $R_{\hat{a}}(\theta_1)$, $\mathrm{tr}\, R_{\hat{a}}(\theta_1) = 0$.

$\therefore \cos\theta_1 = -\frac{1}{2}$ $\boxed{\therefore \theta_1 = 120^\circ}$ ... angle of rotation

Also, $R_{\hat{a}}(\theta) - \tilde{R}_{\hat{a}}(\theta) = 2\,\hat{a}\cdot\vec{J}\sin\theta$. In this case

$$R_{\hat{a}}(\theta_1) - \tilde{R}_{\hat{a}}(\theta_1) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \quad (1)$$