71. DMNR 2

We know that

$$R(\hat{a};\theta) = I + \hat{a}\cdot\vec{J}\sin\theta + (\hat{a}\cdot\vec{J})^2(1-\cos\theta)$$

Also, $\operatorname{tr}(\hat{a}\cdot\vec{J}) = 0$ and $\operatorname{tr}(\hat{a}\cdot\vec{J})^2 = -2\hat{a}\cdot\hat{a}$ $= -2$. Therefore,

a) $\operatorname{tr} R(\hat{a};\theta) = \operatorname{tr} I + \left[\operatorname{tr}(\hat{a}\cdot\vec{J})^2\right](1-\cos\theta) \Rightarrow$

$$\boxed{\operatorname{tr} R = 3 - 2(1-\cos\theta) = 1 + 2\cos\theta}$$

b) Since $\tilde{J}_k = -J_k$, we have

$$\widetilde{(\hat{a}\cdot\vec{J})} = -\hat{a}\cdot\vec{J} \quad \text{and} \quad \widetilde{(\hat{a}\cdot\vec{J})^2} = (\hat{a}\cdot\vec{J})^2.$$

$$\therefore \tilde{R}(\hat{a};\theta) = I - \hat{a}\cdot\vec{J}\sin\theta + (\hat{a}\cdot\vec{J})^2(1-\cos\theta)$$

$$\Rightarrow \boxed{R(\hat{a};\theta) - \tilde{R}(\hat{a},\theta) = 2\hat{a}\cdot\vec{J}\sin\theta}$$