Dragt 70

$$(\hat{a}\cdot\vec{J})^3 \stackrel{?}{=} -(\hat{a}\cdot\vec{J})$$

$\hat{a} = a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3$
is a unit vector so $a_1^2 + a_2^2 + a_3^2 = 1$

$$(\hat{a}\cdot\vec{J}) = a_1 J_1 + a_2 J_2 + a_3 J_3$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -a_1 \\ 0 & a_1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & a_2 \\ 0 & 0 & 0 \\ -a_2 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -a_3 & 0 \\ a_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

$$(\hat{a}\cdot\vec{J})^2 = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -(a_2^2+a_3^2) & a_1 a_2 & a_1 a_3 \\ a_1 a_2 & -(a_1^2+a_3^2) & a_2 a_3 \\ a_1 a_3 & a_2 a_3 & -(a_1^2+a_2^2) \end{pmatrix}$$

$$(\hat{a}\cdot\vec{J})^3 = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}\begin{pmatrix} -a_3^2-a_2^2 & a_1 a_2 & a_3 a_1 \\ a_1 a_2 & -a_3^2-a_1^2 & a_2 a_3 \\ a_3 a_1 & a_3 a_2 & -a_2^2-a_1^2 \end{pmatrix}$$

$$(\hat{a}\cdot\vec{J})^3 = \begin{pmatrix} 0 & a_3(a_1^2+a_2^2+a_3^2) & -a_2(a_1^2+a_2^2+a_3^2) \\ -a_3(\;) & 0 & a_1(\;) \\ a_2(\;) & -a_1(\;) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & a_3 & -a_2 \\ -a_3 & 0 & a_1 \\ a_2 & -a_1 & 0 \end{pmatrix} = -(\hat{a}\cdot\vec{J})$$