a) $$e^{B+C} = \sum_0^\infty \frac{1}{l!}(B+C)^l.$$ But,

$$(B+C)^l = \sum_{p=0}^{l} \binom{l}{p} B^{l-p} C^p \quad \boxed{\text{if } BC = CB}.$$ Therefore,

$$e^{B+C} = \sum_{l=0}^\infty \frac{1}{l!} \sum_{p=0}^{l} \binom{l}{p} B^{l-p} C^p = \sum_{\substack{m=0\\n=0}}^\infty \frac{1}{(m+n)!}\binom{m+n}{n} B^m C^n.$$

Here the sum has been reordered by setting $l-p=m$, $p=n$.

But, $$\frac{1}{(m+n)!}\binom{m+n}{n} = \frac{1}{m!}\,\frac{1}{n!}.$$ Therefore,

$$e^{B+C} = \sum_{m=0}^\infty \frac{1}{m!} B^m \sum_{n=0}^\infty \frac{1}{n!} C^n = e^B e^C \text{ if } [B,C]=0.$$

b) $$e^{-B} e^{B} = e^{-B+B} = e^0 = I.$$ Note that $[-B, B] = 0$

c) $$e^{CBC^{-1}} = \sum_0^\infty \frac{1}{n!}(CBC^{-1})^n$$

$$= \sum_0^\infty \frac{1}{n!} C B^n C^{-1} = C e^B C^{-1}.$$