

We need $b (b^2 + \kappa)^{\frac{3}{2}}$ in terms of Θ_S . Since $(1 - \frac{\Theta_S}{\pi}) = \frac{b}{(b^2 + \kappa)^{\frac{1}{2}}}$,

$$(1 - \frac{\Theta_S}{\pi})^3 = \frac{b^3}{(b^2 + \kappa)^{\frac{3}{2}}} \Rightarrow b (b^2 + \kappa)^{\frac{3}{2}} = \frac{b^4}{(1 - \frac{\Theta_S}{\pi})^3}$$

We also have $b^2 = (b^2 + \kappa) (1 - \frac{\Theta_S}{\pi})^2$

$$\Rightarrow b^2 [1 - (1 - \frac{\Theta_S}{\pi})^2] = \kappa (1 - \frac{\Theta_S}{\pi})^2 \Rightarrow b^4 = \frac{\kappa^2 (1 - \frac{\Theta_S}{\pi})^4}{[1 - (1 - \frac{\Theta_S}{\pi})^2]^2}$$

$$\Rightarrow \sigma = \frac{1}{\pi \kappa} \frac{1}{\sin \Theta_S} \frac{\kappa^2 (1 - \frac{\Theta_S}{\pi})^4}{[1 - (1 - \frac{\Theta_S}{\pi})^2]^2} \frac{1}{(1 - \frac{\Theta_S}{\pi})^3} = \frac{\kappa}{\pi} \frac{1}{\sin \Theta_S} \frac{(1 - \frac{\Theta_S}{\pi})}{(\frac{2\Theta_S}{\pi} - \frac{\Theta_S^2}{\pi^2})^2}$$

Putting in value for $\kappa \Rightarrow$

$$\sigma = \frac{\kappa}{m v_0^2} \frac{(1-x)}{x^2 (2-x)^2 \sin \pi x} \frac{1}{\pi} \quad \text{with } x = \frac{\Theta_S}{\pi}, \quad m v_0^2 = 2E$$

$$\sigma_{\text{Gold}} = \frac{\kappa}{2E} \frac{(1-x)}{x^2 (2-x)^2 (\sin \pi x)}$$

Which agrees with Goldstein.