Goldstein 3.31

We have $f = k r^{-3}$, repulsive, $\Rightarrow V(r) = \frac{1}{2} k r^{-2}$, $k > 0$.

Let b = impact parameter + Θ_s = scattering angle. Then Goldstein

(3-93) $\Rightarrow$ $\boxed{\sigma(\Theta_s) = -\frac{b}{\sin\Theta_s}\frac{db}{d\Theta_s}}$.

Starting at ∞, the initial conditions for the orbit are $p_\theta = m v_0 b$ + $E = \frac{1}{2} m v_0^2$ where v_0 = initial velocity.

Looking at Goldstein (3.36) $\Rightarrow$

$\Theta_s = \pi - \Delta$ with

$$\Delta = 2\int_{r_{min}}^{\infty} \frac{p_\theta \, dr}{r^2\sqrt{2m(E - V_{eff})}} \quad \text{with } V_{eff} = \frac{1}{2} k r^{-2} + p_\theta^2 (2m)^{-1} r^{-2}$$

Do some algebra $\Rightarrow$

$$\Delta = 2\int_{r_{min}}^{\infty} \frac{m v_0 b \, dr}{r^2\sqrt{2m\left(\frac{1}{2} m v_0^2 - \frac{m^2 v_0^2 b^2}{2 m r^2} - \frac{k}{2 m r^2}\right)}}$$

$$= 2\int_{r_{min}}^{\infty} \frac{b \, dr}{r^2\left\{1 - \frac{b^2}{r^2} - \frac{K}{r^2}\right\}^{\frac{1}{2}}} \quad \text{where } K = k/(m v_0^2)$$

$$= 2b\int_{r_{min}}^{\infty} \frac{dr}{r\left\{r^2 - (b^2+K)\right\}^{\frac{1}{2}}} = 2b\frac{1}{\sqrt{b^2+K}}\cos^{-1}\frac{(b^2+K)^{\frac{1}{2}}}{r}\Bigg|_{r_{min}}^{\infty} = \frac{\pi b}{\sqrt{b^2+K}}$$

$$\therefore \Theta_s = \pi\left(1 - \frac{b}{\sqrt{b^2+K}}\right) \Rightarrow \frac{d\Theta_s}{db} = \frac{-\pi K}{(b^2+K)^{3/2}} \Rightarrow \boxed{\sigma = \frac{1}{\pi K}\frac{1}{\sin\Theta_s} b(b^2+K)^{3/2}}$$