Precessional 'rate' = amount of precession in one "period" (θ going from 0 to 2π since perihelion = where r min, ie where $\alpha\theta = 0, 2\pi, \ldots$)

$$\Delta\theta_p = 2\pi - \frac{2\pi}{\alpha}$$

or

$$\Delta\theta_p = 2\pi\left(\frac{\alpha-1}{\alpha}\right). \quad \text{But} \quad \alpha = \left(1+\frac{mB}{\ell^2}\right)^{\frac{1}{2}} \simeq 1 + \frac{mB}{2\ell^2}$$

$$\Rightarrow \Delta\theta_p \simeq \frac{2\pi mB}{2\ell^2} = \frac{\pi mB}{\ell^2}.$$ Dividing by the period τ gives

$$\boxed{\dot{\Omega} = \frac{\Delta\theta_p}{\tau} = \frac{\pi mB}{\ell^2\tau} = \frac{2\pi mh}{\ell^2\tau}}$$

Also,

$$\Delta\theta_p = \frac{\pi mB}{\ell^2} = \frac{2h}{ka}\,\frac{\pi mka}{\ell^2} = 2\eta\pi\left(\frac{km}{\ell^2}\right)a = 2\eta\pi\left(\frac{1}{\beta}\right)\frac{\beta}{1-\epsilon^2}$$

$$\therefore \boxed{\Delta\theta_p = \frac{2\pi\eta}{1-\epsilon^2} \quad \text{or} \quad 2\eta = \frac{1-\epsilon^2}{\pi}\Delta\theta_p}$$

For Mercury $T = .24$ yr

$\Delta\theta_p \rightarrow 40''$ arc/century $= .096$ second arc/period

$\epsilon = .206$

$$2\eta = \left(\frac{1-\epsilon^2}{\pi}\right)\left(\frac{.096}{3600}\,\frac{\pi}{180}\right)$$

To convert to degrees ↑ ↑ To convert to radians

$$\Rightarrow \boxed{2\eta = \frac{(.96)(.096)}{(180)(3600)} = 1.42\times10^{-7}}$$

$$\boxed{\text{and } \eta = 7.1\times10^{-8}}$$