Let $V(r) = -k/r + \frac{h}{r^2}$

Then $f(r) = -V'(r) = -\frac{k}{r^2} + \frac{B}{r^3}$ with $\boxed{B = 2h}$

The orbit equation becomes

$$\frac{d^2u}{d\theta^2} + u = \frac{-m}{l^2u^2}\left(-ku^2 + Bu^3\right)$$

or $$\frac{d^2u}{d\theta^2} + \left(1 + \frac{mB}{l^2}\right)u = \frac{mk}{l^2}$$

The solution to which is, of course

$$u(\theta) = \frac{mk}{l^2} + A\cos\left(\sqrt{1 + \frac{mB}{l^2}}\,\theta\right)$$

Phase arb. picked to be 0.

$$= \frac{1}{\beta} + \frac{\varepsilon}{\beta}\cos\left(\sqrt{1 + \frac{mB}{l^2}}\,\theta\right)$$

So $$r = \frac{\beta}{1 + \varepsilon\cos\alpha\theta}$$

with $\beta = l^2/km$ & $\alpha = \sqrt{1 + \frac{mB}{l^2}}$ and ε

determined by initial conditions. If we let $a = \beta(1-\varepsilon^2)^{-1}$ we get the result

$$r = \frac{a(1-\varepsilon^2)}{1 + \varepsilon\cos\alpha\theta}.$$

This is an ellipse for $\alpha = 1$, but is a <u>precessing</u> ellipse if $\alpha \neq 1$.