we arrive at

$$t_f = 2 r_0^{3/2} \left(\frac{m}{2k}\right)^{\frac{1}{2}} \int_0^1 \frac{u^2\,du}{\sqrt{1-u^2}}$$

we've already seen this integral in the Brachistochrone problem

$$\int_0^1 \frac{u^2\,du}{\sqrt{1-u^2}} = \left(-\frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\sin^{-1} u\right)\Big|_0^1$$

$$= \frac{\pi}{4}.$$

$$\boxed{t_f = \frac{\pi}{2} r_0^{3/2} \left(\frac{m}{2k}\right)^{\frac{1}{2}}}$$

Now use the 1st baloned result to get

$$t_f = \frac{\pi}{2} \frac{\tau}{2\pi} \left(\frac{k}{m}\right)^{\frac{1}{2}} \left(\frac{m}{2k}\right)^{\frac{1}{2}} = \frac{\tau}{4\sqrt{2}}. \quad (!)$$