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we arrive at

$$t_f = 2r_0^{3/2} \left(\frac{m}{2k}\right)^{1/2} \int_0^1 \frac{u^2 du}{\sqrt{1-u^2}}$$

we've already seen that integral in the Brachistochrone problem

$$\begin{aligned} \int_0^1 \frac{u^2 du}{\sqrt{1-u^2}} &= \left( -\frac{u}{2} \sqrt{1-u^2} + \frac{1}{2} \sin^{-1} u \right) \Big|_0^1 \\ &= \frac{\pi}{4} . \end{aligned}$$

$$t_f = \frac{\pi}{2} r_0^{3/2} \left(\frac{m}{2k}\right)^{1/2}$$

Now use the 1st ballooned result to get

$$t_f = \frac{\pi}{2} \frac{\tau}{2\pi} \left(\frac{k}{m}\right)^{1/2} \left(\frac{m}{2k}\right)^{1/2} = \frac{\tau}{4\sqrt{2}} . (!)$$