Goldstein 3.11

$$T = \frac{1}{2} m r^2 \dot{\theta}^2$$ (Kinetic Energy)

But the Virial Thm gives us $T = -\frac{1}{2} V$

$= + \frac{k}{2r}$

So $m r^2 \dot{\theta}^2 = \frac{k}{r}$ or, since $\dot{\theta} = \frac{2\pi}{\tau}$

$$r_0^3 = \frac{k}{m} \frac{\tau^2}{(2\pi)^2}$$

$$\boxed{r_0^{3/2} = \left(\frac{k}{m}\right)^{\frac{1}{2}} \frac{\tau}{2\pi}}$$

This relates radius & period during the orbit.

Now $t_f = \left(\frac{m}{2}\right)^{\frac{1}{2}} \int_0^{r_0} \frac{dr}{[E - V(r)]^{1/2}}$ (3.27)

Note: r is relative inter particle coordinate.

But the particles are stopped at r_0 so

$$t_f = \left(\frac{m}{2}\right)^{\frac{1}{2}} \int_0^{r_0} \frac{dr}{[V(r_0) - V(r)]^{1/2}}$$

$$= \left(\frac{m}{2}\right)^{\frac{1}{2}} \int_0^{r_0} \frac{dr}{\left[\frac{k}{r} - \frac{k}{r_0}\right]^{\frac{1}{2}}}$$

$$= r_0^{3/2} \left(\frac{m}{2k}\right)^{\frac{1}{2}} \int_0^1 \frac{(r/r_0)^{1/2}\, d(r/r_0)}{[1 - r/r_0]^{1/2}}$$

Letting $r/r_0 = u^2 \Rightarrow d(r/r_0) = 2u\,du$

so that $(r/r_0)^{\frac{1}{2}}\, d(r/r_0) = 2u^2\,du$