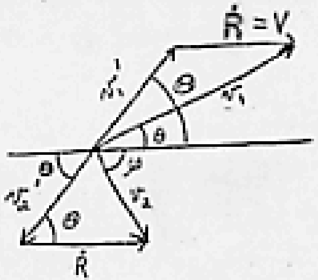
Goldstein 3.7

Completing fig. 3. for the scattered particle and target:



where v' refers to velocity in the CM.

We know that in CM: $m_1 v_1' = m_2 v_2'$ where

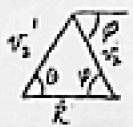
$$v_1' = \frac{\mu}{m_1} v_0 = \frac{m_2}{m_1 + m_2} v_0$$

The CM velocity $\dot{R}$ is $(\dot{R} = V)$

$$\dot{R} = \frac{\mu}{m_2} v_0 = \frac{m_1}{m_1+m_2} v_0$$

So $v_2' = \frac{m_1}{m_2} v_1' = \frac{m_1}{m_2} \frac{m_2}{m_1+m_2} v_0 = \frac{m_1}{m_1+m_2} v_0 = \dot{R}$

That is, in the CM system, the recoil speed is equal in magnitude to the CM speed.



The triangle is isosceles ($v_2' = \dot{R}$) so the angle between v_2' and v_2 must also be φ, and thus,

$$\varphi + \varphi + \Theta = \pi$$

or $\varphi = \frac{1}{2}(\pi - \Theta)$

G 3.7

~~G 3.7~~ cont.

b) Suppose the scattering is isotropic in terms of Θ. We take this to mean that $\sigma(\Omega)$ in the center of mass frame is independent of Θ. In the language of quantum mechanics, this is equivalent to saying that the scattering is purely S-wave.

Let $\chi(a; b, \Delta b)$ be the "characteristic function" for the interval $(b, b+\Delta b)$.

It is defined by the requirement

$$\chi(a; b, \Delta b) = 1 \quad \text{for } a \in (b, b+\Delta b)$$
$$= 0 \quad \text{for } a \notin (b, b+\Delta b).$$

It has the graph shown below.