64. cont.

Show that this differential equation has the unique solution

$$f(\lambda) = \exp(\lambda \mathrm{tr}(B)].$$

From ★★ we have

$$\frac{df}{f} = d\lambda \, \mathrm{tr}(B) \Rightarrow$$

$$\log[f(\lambda')]\Big|_0^\lambda = \lambda \, \mathrm{tr}(B) \Rightarrow$$

$$\log[f(\lambda)] - \underbrace{\log[f(0)]}_{= \log[1] = 0} = \lambda \, \mathrm{tr} B$$

$$\Rightarrow f(\lambda) = e^{\lambda \, \mathrm{tr} B} \quad ★★★$$

Consequently, show that if A and B are any two matrices related by (7.4), then

$$\det(A) = \exp[\mathrm{tr}(B)]. \qquad (3.7.96)$$

Putting $\lambda = 1$ in ★★★ gives the desired result.