64.

7.7: Suppose that ϵ is a small parameter and B is an arbitrary matrix. Verify the expansion

$$\det(I+\epsilon B) = 1 + \epsilon\,\mathrm{tr}(B) + O(\epsilon^2). \tag{3.7.95}$$

If we expand $\det(I+\epsilon B)$ by minors and keep only terms constant and linear in ϵ, what do we get? Consider the 3×3 case:

$$\det(I+\epsilon B) = \begin{vmatrix} 1+\epsilon B_{11} & \epsilon B_{12} & \epsilon B_{13} \\ \epsilon B_{21} & 1+\epsilon B_{22} & \epsilon B_{23} \\ \epsilon B_{31} & \epsilon B_{32} & 1+\epsilon B_{33} \end{vmatrix}$$

$$= (1+\epsilon B_{11}) \begin{vmatrix} 1+\epsilon B_{22} & \epsilon B_{23} \\ \epsilon B_{32} & 1+\epsilon B_{33} \end{vmatrix} + O(\epsilon^2)$$

$$= (1+\epsilon B_{11})(1+\epsilon B_{22})(1+\epsilon B_{33}) + O(\epsilon^2)$$

$$= 1 + \epsilon\left[B_{11} + B_{22} + \epsilon B_{33}\right] + O(\epsilon^2)$$

$$= 1 + \epsilon\,\mathrm{tr}\,B + O(\epsilon^2).$$

We see that only the product of the diagonal terms contribute.