Thus, the lens cap is <u>behind</u> the cosmonaut at $t = \tau$ even though it is going <u>faster</u>!

On the other hand, if the lens cap is thrown against the orbit velocity, so that it is going more slowly than the cosmonaut, then it arrives back at $\phi = 2\pi$ <u>before</u> the cosmonaut, and he sees it ahead of him when he arrives at $\phi = 2\pi$!

g) Suppose the lens cap is thrown out of the plane of the orbit. Let r, ϕ, z be cylindrical coordinates. Then

$$L = \left(\tfrac{1}{2}\right) m \left[\dot{r}^2 + (r\dot{\phi})^2 + \dot{z}^2\right] + \frac{m M G}{\left[r^2 + z^2\right]^{1/2}} .$$

For small excursions out of the equatorial plane, $z \sim \epsilon$, and

$$\left[r^2 + z^2\right]^{-1/2} = \left[r^2 (1 + z^2/r^2)\right]^{-1/2} = r^{-1} \left(1 - \frac{z^2}{2r^2}\right) + \cdots$$

$$= r^{-1} - \frac{z^2}{2R^3} + O(\epsilon^3)$$