These conditions can be be met by setting $\gamma = 0$, $\beta = -\frac{2R\alpha}{\omega}$. This gives

$$\boxed{\rho = \frac{2R\alpha}{\omega}\left[1 - \cos\omega t\right]}$$

Also, set $\delta = 0$. This gives

$$\boxed{\psi = -3\alpha t + \frac{4\alpha}{\omega}\sin\omega t}$$

Note that now $\dot{\psi}(0) = -3\alpha + 4\alpha = \alpha$.

Evaluating ρ and ψ at $t = \tau = 2\pi/\omega$ gives

$$\boxed{\begin{aligned} \rho(\tau) &= 0 \\ \psi(\tau) &= -3\alpha(2\pi/\omega) = -6\pi\frac{\alpha}{\omega} \end{aligned}}$$

Observe that if $\dot{\psi}(0) = \alpha > 0$, then

$\psi(\tau) = -6\pi\frac{\alpha}{\omega} < 0$. Therefore,

$\phi(\tau) = \omega\tau + \epsilon\,\psi(\tau) = 2\pi - \epsilon\, 6\pi\frac{\alpha}{\omega} < 2\pi$