a) $L = (1/2)\, m[\dot{r}^2 + (r\dot{\phi})^2\,] + mMG/r$ (A1)

b) $\ddot{r} = -MG/r^2 + r\dot{\phi}^2$ (A2)

$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0$ (A3)

c) $\omega^2 = MG/R^3$ (A4)

d) $\ddot{\rho} = 3\omega^2\rho + 2\omega R\dot{\psi}$ (A5)

$\ddot{\psi} + 2(\omega/R)\dot{\rho} = 0$ (A6)

e) Integrating (A6) and using initial conditions gives

$$\dot{\psi} + 2(\omega/R)\rho = 0\ . \qquad \text{(A7)}$$

Putting this in (A5) gives

$$\ddot{\rho} = -\omega^2\rho\ .$$

Taking into account the initial condition, the solution for ρ is

$$\rho = -\frac{v}{\varepsilon\omega}\sin\omega\, t\ . \qquad \text{(A8)}$$

Correspondingly, ψ can be found by putting (A8) in (A7) and integrating. The result is

$$\psi = \frac{2v}{\varepsilon\omega R}\,(1 - \cos\omega\, t). \qquad \text{(A9)}$$

Note that when $t = \tau = 2\pi/\omega$ (one period later), then $\rho(\tau) = \psi(\tau) = 0$. Also

$$\dot{\psi}(\tau) = -2(\omega/R)\,\rho(\tau) = 0.$$