putting $t = t^f$ and using (4.57) gives (4.59). 5/5

60 cont.

From this result show that the determinant of the Jacobian matrix associated with any transfer map arising from a real differential equation must satisfy the condition

$$\det(M) > 0. \tag{1.4.60}$$

Geometrically, this condition means that $\mathcal{M}$ preserves orientation. For example, in the case $m = 3$, the $\mathcal{M}$ arising from any real differential equation cannot send a right-handed triad into a left-handed triad.

We know that $e^x > 0$ if x is real and finite. Suppose that A is real (which it will be for real differential equations) and that $\operatorname{tr} A$ is integrable. Then we will have

$$\int_{t^i}^{t^f} dt\, \operatorname{tr}[A(t)] \text{ real and finite,}$$

and hence

$$\det(M) = \exp\{\int_{t^i}^{t^f} dt\ \operatorname{tr}[A(t)]\} > 0.$$