60 cont.

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$$\epsilon \dot{\eta}_j = \epsilon \sum_k A_{jk} \eta_k + O(\epsilon^2) \quad \star\star.$$

Dividing $\star\star$ by ϵ gives

$$\dot{\eta}_j = \sum_k A_{jk} \eta_k + O(\epsilon) \quad \star\star\star.$$

Now letting $\epsilon \to 0$ gives (1.4.50).

These are the variational equaitons associated with (4.1) around the trajectory y^d. Note that they are *linear* even if (4.1) is nonlinear. Let $L(t)$ be the $m \times m$ matrix defined by the *matrix* differential equation (a collection of m^2 ordinary differential equations)

$$\dot{L} = A(t)L \tag{1.4.51}$$

with the initial condition

$$L(t^i) = I \tag{1.4.52}$$

where I denotes the $m \times m$ identity matrix. Show that the solution to (4.50) with the initial condition η^i is given by the prescription

$$\eta(t) = L(t)\eta^i. \tag{1.4.53}$$

Evidently from (4.52) and (4.53) we have

$$\vec{\eta}(t^i) = L(t^i)\vec{\eta}^i = I\vec{\eta}^i = \vec{\eta}^i.$$

Also, differentiating (4.53) and using (4.51) gives

$$\dot{\vec{\eta}} = \dot{L}\vec{\eta}^i = AL\eta^i = A\vec{\eta},$$

and so (4.50) is satisfied.

Show that the desired Jacobian matrix is given in terms of $L(t)$ by the relation

$$M = L(t^f). \tag{1.4.54}$$