$$\boxed{\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{2E}{L^2} + \frac{2M_S G}{L^2}u}$$ Newton.

So, General Relativity gives the extra term $2M_S G c^{-2} u^3$! Let us write the Relativistic result in the form of eqn 1:

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{2}{L^2}\left(E + M_S G u + \frac{M_S G L^2}{c^2} u^3\right)$$

Compare with (see notes, Eq. 1)

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{2}{L^2}\left(E - F(u)\right) \Rightarrow$$

$$\boxed{F(u) = -M_S G u - M_S G L^2 c^{-2} u^3}$$

Now use formulas on page 3 of notes:

$$F'(u) = -M_S G - 3 M_S G L^2 c^{-2} u^2$$

$$F''(u) = -6 M_S G L^2 c^{-2} u$$

$$\left\{1 - u F'(u)/F'(u)\right\} =$$

$$1 - 6 M_S G L^2 c^{-2} u^2/(M_S G + 3 M_S G L^2 c^{-2} u^2)$$

$$= 1 - 6 L^2 c^{-2} u^2/(1 + 3 L^2 c^{-2} u^2)$$